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SELECTION OF MACHINE TOOLS OPTIMAL CUTTING MODES FOR DESIGNERS

Monograph

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The monograph discusses the problems of finding optimal cutting modes for the operation of turning, drilling and milling on machines of various types. Formulations of optimization problems in single and multi-criteria versions are given. Methods, algorithms and programs for solving problems of choosing optimal cutting conditions are proposed. The application features of the geometric programming methods by zero and first degree of difficulty are considered and the area of their rational use is defined. A toolkit for solving nonlinear technological problems by the method of Lagrange multipliers for cases of two- and three-pass treatments has been developed. Models and algorithms for solving multicriteria problems by methods of criteria convolution, consecutive climb-down and the "ideal point" are created.

For leading designers and technologists, specialists in the field of computer-aided design of technological systems, scientists, lecturers, graduate students and students.

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INTRODUCTION

The widespread introduction of new structural and instrumental materials, high-performance CNC equipment and adaptive control systems, involves the use of developed mathematical methods and software for the design of technological systems. Successful implementation of these requirements is associated with the application of knowledge to optimize design decisions and to control the cutting process in real-time.

A modern mechanical engineer must be familiar with the methodology and algorithms for solving problems of choosing optimal cutting conditions. This becomes especially relevant when operating expensive software-controlled equipment. But the optimal cutting modes are assigned based on knowledge of physics, the cutting process, mathematical modeling of the problem, and the use of computational search algorithms, to which this monograph is dedicated.

No less important are the issues of controlling the cutting process. The full range of tasks related to the design and architecture of control systems of various classes and the preparation of control programs for CNC machine tools and machining centers should be in the arsenal of a modern specialist.

It is difficult to overestimate the role of computational methods for choosing the best options for solving various engineering and economic problems, which allows in many cases to apply strict formal solutions instead of approximate solutions or complex experiments and use them in the automatic control of production processes.

At the same time, the successful use of computer toolkit and software to automate the management of production processes and engineering activities gives an effect only when a comprehensive study

and mathematical description of technological processes to be automated, and therefore the creation of mathematical models of these processes. There is a need for research to create and improve the implementation algorithms of these mathematical models using modern computer technology.

Calculation and selection of optimal cutting conditions are one of the special tasks in the process of designing manufacturing technology for products in mechanical engineering, special and modular machines, as well as multi-operation equipment with CNC. At the same time, determining the optimal cutting conditions is an important part of the process based on which the development of the entire project is carried out. So, based on a preliminary assessment of the limiting processing conditions, the development of:

- kinematics of the machine - rotational speeds, feed rates, a partition of allowance for machining pass;
- power drives, including electric motor power, forces arising during cutting, torque values on spindles and machine shafts, strength and rigidity of forming units;
- dynamics of the machine – eigenfrequencies, vibration modes, transfer functions of the machine elastic links.

Based on the calculated cutting conditions, the frequency and order of change of cutting tools, tool life is established, the number of workers, the fleet of metal cutting machines necessary to carry out the given program, the necessary investments and many other elements of the organization and economy of production are determined.

Based on these considerations, the right choice of truly optimal cutting conditions that provide the greatest economic effect, taking into account the maximum possible number of factors affecting productivity, technical quality, and production economics, is of great importance.

The subject of this monograph is the processes and algorithms for adopting optimal modes of metal cutting, based on the developed mathematical apparatus and modern mathematical environments such as Maple.

The purpose of this kind of research is to improve the procedures for technological preparation of production and provide designers and technologists with detailed methods for making optimal decisions.

To achieve this goal put forward several objectives of this study:

- analysis and selection of optimality criteria in the calculation of cutting conditions;
- the formation of a system of technical constraints focused on the specifics of the designed equipment and process;
- development of methods and algorithms for scalar (single-criterion) optimization;
- development of methods and algorithms for multicriteria (vector) optimization;
- Implementation of optimization procedures for cutting conditions in various conditions of machine-building production.

The author is sincerely grateful to Professor G. Khmelovsky. for his creative approach to solving problems of optimizing technological systems, for his contribution to the creation of analytical tools for finding optimal cutting conditions.

1. OPTIMIZATION OF THE MACHINING PROCESS FOR ENGINEERING PRODUCTS

1.1. The main approaches to solving problems optimization of the machining process

To date, various approaches to solving optimization problems in engineering products have been formed. An analysis of approaches to the calculation of cutting conditions and technical norms of time used at enterprises showed that the latter almost all use the collection “General machine-building norms of cutting regimes for technical regulation of work on metal-cutting machines” [1].

The use of centrally developed standards at enterprises allows to unify the purpose of norms and regimes and is characterized by an approximately equal level of tension of norms at different enterprises. At the same time, many enterprises use several new processed materials and cutting tools. The development of standards for machining in these cases is carried out by factory regulatory research bureaus. It should be borne in mind that general engineering standards provide only an approximate level of standards of equal tension.

Much depends on the correct use of the standards. In general engineering standards [1], certain decisions correspond to the initial conditions for their choice. When developing algorithms for calculating cutting conditions, it is difficult to find a match between the calculation results and the source data relating to the developed technological process. Therefore, in the developed algorithms, intermediate conditionally constant information is often taken into account, reflecting the specifics of certain production conditions. At the same time, making changes to such algorithms and programs is difficult.

Some existing work on optimizing cutting conditions focuses on the study of the influence of variables on the cutting process using empirical dependencies in the tasks of increasing the productivity and durability of cutting tools, as well as the productivity of machines with given requirements for accuracy and surface roughness of parts. At one time, the work of [2–5] and other researchers in the field of metal cutting was devoted to the development of empirical formulas.

Several authors on optimization of cutting conditions use previously created and formulate new empirical formulas that reflect the dependence of variables affecting the cutting process [6–8]. An analysis of these and other works in this field showed that it is difficult to find such analytical formulas that will reflect the interdependence of all factors affecting cutting.

When choosing cutting modes based on empirical formulas, it is also not possible to identify the relationships between many influencing factors, such as vibration, lubrication fluids oil-based (e.g. MHA ISO 6743-0-81), geometric shapes of cutting inserts, etc. In this regard, the calculation of cutting modes according to empirical formulas was acceptable and quite effective for a limited range of metal cutting machines and a limited range of materials for cutting tools and processed materials.

In works on the optimization of cutting conditions [9, 10], the concepts of the resistance of the Cutting Tool (CT) are used, which ensures the highest productivity of the machine $T_{pr.max}$ (tool life of the highest productivity) and the tool life of CT, which ensures the lowest cost price T_{cl} (economic tool life) To determine the values of $T_{pr.max}$ and T_{cl}, the well-known productivity formulas P are used as a function of the piece time t_{pc} and the variable part of the cost price C_v. As a result of successive transformations and the presentation of these equations as

functions of the tool life period T, the values of $T_{pr.max}$ and T_{cl} are determined by taking the first derivative of these equations for T and equating it to zero.

An analysis of the results shows the dependence of the optimum resistance on the value of μ – the value inverse to the relative tool life and to the tool change time. Moreover, the tool life of the lowest cost is always greater than the tool life of the highest productivity by the amount of the reduced costs of manufacturing and the tool operational phase. For the CT tool life thus calculated, it is possible to choose many combinations of cutting depth d, feed f and cutting speed V. The most favorable cutting mode is considered by the authors [9, 10] as a mode that ensures the lowest cost of machining provided that all requirements for product quality and given machine productivity are satisfied. Considering the tool life as a constant value to achieve minimum machine time (maximum productivity), the following sequence of the procedure for determining cutting conditions was proposed [9, 10]:

1) the choice of cutting tools;
2) setting the depth of cut;
3) determination of feed;
4) determination of V_T, providing the most favorable period of tool life.

A similar approach in the 2011 textbook [11] and the 2013 study guide [12] is presented.

In [11], a situation is considered when the accuracy of the workpiece being machined, which allows cutting in one pass, is set and the cutting depth d is taken to be equal to the machining allowance. In this case, two possible solutions are proposed:

1. Set the maximum possible feed f, admissible by the quality of the manufactured part and technical constraints, and find the value of the cutting speed V_o, which corresponds to the adopted optimization criterion.

2. Find a combination of feed and cutting speed that corresponds to the accepted optimization criterion.

In the textbook [11], the optimization procedure for cutting conditions is described in detail both in the monotonous and non-monotonous sections of the *T-V* tool life relationship.

In those cases when there are no inflection points on the curve $T = f(V)$, i.e. when, with an increase in speed, the tool life of CT continuously decreases, the optimization procedure for cutting conditions is implemented by forming a criterion of technological cost price *C*, taking the first derivative to speed and equating it to zero. As a result of this, the optimum value of the cutting speed V_{Cmin} is determined as a function of the relative tool life index m_v and tool change time t_{ch}. With this value of speed, the economic period of tool life T_{cp} is determined, with a decrease in which, the corresponding speed V_{Cmin} will increase, and, consequently, the productivity of machining will increase.

A similar procedure can be used to search for the speed V_{prmax} according to the criterion of maximum productivity and the corresponding value of the tool life period with the highest productivity T_{prmax} [11]. Similarly to [9, 10], it is concluded that the resistance period of the highest productivity is less than the economical resistance period $T_{prmax} < T_{Cmin}$, and the cutting speed $V_{pr\max}$ is greater than the speed V_{Cmin}.

A different situation arises when machining steels with a carbide tool, when the tool life dependence is described by a curve on which there are extreme values (maximum and minimum) and inflection points, i.e., in the presence of a nonmonotonic relationship. A section of such a curve is approximated by a combination of power and exponential functions and is expressed by the dependence [2, 5]

$$T = \frac{C_{T0} V^k}{e^{k_1 v} t^p S^q}.$$

For a given feed and cutting depth, the optimal value of V_{Cmin} is determined by solving a transcendental equation that relates the technological cost and cutting speed, and after substituting the obtained value in the tool life dependence, the economic tool life period is determined. Finding the root of the transcendental equation using numerical methods also allows us to find the optimal value of V_{prmax} and the tool life T_{prmax} that meets the criterion of maximum productivity.

In addition to the two most common optimization criteria: minimum cost price and maximum productivity, the authors also consider the criterion for the maximum resource *Q*. The maximum cutting speed V_{Qmax} and the corresponding durability period T_{Qmax} are calculated using this criterion using the procedure for finding the optimal solution described above.

The authors of the textbook constructed graphs of the influence of cutting speed not the resistance period, tool life, technological cost and productivity both for the monotonic and non-monotonic nature of the *T–V* connection (with an index of tool life relative $m_v = 0.27$). The graphs show that with the extreme nature of the dependence $T = f(V)$, resource with increasing speed initially grows, passing through a maximum, and then decreases. It should be noted that the maximum tool life does not correspond to the maximum resource period: $V_{Qmax} > V_{Tmax}$ and $V_{Cmin} > V_{Qmax}$.

Along with one-parameter optimization, the textbook also considers two-parameter optimization (optimized variables – feed and cutting speed) according to the criteria of minimum cost price, maximum productivity and maximum resource. The optimization procedure is carried out according to the following algorithm:

- the formation of analytical expressions for the above optimization criteria with the substitution of the expression of the tool life period as a function of cutting speed *V* and feed *f*;
- construction in a 3D surface $C = f(V, f)$;
- the joint solution of the equations $\partial C/\partial V = 0$ and $\partial C/\partial f = 0$ by numerical methods and finding the optimal combination of *V* and *f*.

An analysis of the results shows that the speeds that provide the minimum cost price and maximum productivity are equal. The feed corresponding to the maximum productivity is greater than the feed corresponding to the minimum cost price. Moreover, we have a ratio between the periods of resistance: $T_{prmax} < T_{Cmin}$. Substituting the expression of the tool life index into the tool resource formula allows evaluating the effect of increasing the feed, leading to a decrease in the resource criterion. In this case, the cutting speed affects resource non-monotonously.

Along with technical and economic criteria, optimization works also use technological criteria [13], such as cutting intensity (*R*) and tool life *T*, which are implicitly related to the economic performance of the cutting process. It is especially important to operate with these criteria when there is no data necessary for estimation of price cost and productivity. For such a multicriteria problem, an admissible cutting mode is determined that has the property that there is no other admissible mode that provides a simultaneous increase in the parameters *R* and *T* compared with those values, that correspond to the optimum mode. The author [13] uses the characteristic functions “cutting intensity – tool life” *R*–*T*–*F*, which have the properties of describing all possible extreme combinations of the parameters *R* and *T* for a given machining process.

An interesting approach to the optimization problem is proposed in the works of G.Yu. Jacobs and his co-authors [14], who present this task as two-level:

- at the 1_{st} level, the found parameters of the cutting process are considered as constant values, i.e. at this stage, a statistical interpretation of the cutting process is carried out, without taking into account the disturbing influences changing over time;

- at the 2_{nd} level, adaptive control of the cutting process is carried out, which consists of targeted regulation of the process to maintain the constancy of its most important technical and economic parameters. At this level, a dynamic interpretation of the process is implemented, taking into account the perturbing influences changing over time.

This approach is associated with the formation of various mathematical models (objective functions and technical constraints) depending on the optimized values and the given parameters of the technological task in the process of manufacturing engineering products. This provides an optimal combination of operating modes.

In the process of technological design, various power and other nonlinear dependencies are widely used, which describe the nature of the change in kinematic, power, tool life, and other parameters. An effective optimization method in the presence of nonlinearities in the objective function and constraints is the method of geometric programming (GP) [15; 16].

The geometric programming method is a conditional optimization method that is used to solve problems, where the left-hand sides of the constraints and the optimization criterion are polynomial functions of the form:

$$f(x) = \sum_{t=1}^{T} C_t \Pi x_n^{\alpha_{nt}},$$

called posynomials with *N* variables and *T* members; C_t –positive constants; $x = (x_1, x_2, ..., x_n)^T$ – vector of variables; α_{nt} – any real number.

In the works of scientists of DonNTU [17-19], the GP method was used for various options of fine, precision turning and boring of constructional materials with a tool with superhard material (SHM) plates. The minimum cost price (the variable part of the machining cost price carried out in one pass) and the maximum productivity with constraints on the tool cutting capabilities, the allowable roughness of the machined surface of the part and the maximum temperature in the cutting zone are considered as optimization criteria. The cutting speed and feed are considered as optimized variables, and the cutting depth is equal in value to the machining allowance. Using the GP method, the authors obtained some dependencies for determining the optimal cutting conditions and plotted the dependences of the optimal modes on the temperature and the surface roughness level. These analytical dependencies allow you to control the cutting process for various machining conditions, which increases the efficiency of production technology. Besides, based on these dependencies, the coefficients of changes in the cost price and productivity were calculated with a deviation of the cutting conditions from the optimal values, which is especially important for cases of machining parts on metal-cutting machines with step regulation.

For the case of the precision turning of cast irons using materials from SHM, numerical relationships between pairs of optimal cutting conditions are found. So, when operating in modes that ensure the minimum cost price, productivity losses are 25%, and when working with maximum productivity, cost price losses do not exceed 5%.

Features of choosing the most advantageous modes multi-tool cutting

The procedure for selecting modes is carried out in the following sequence:

1. Formation of the efficiency criterion – the lowest cost price or greatest productivity.

2. Assignment of the greatest depth of cut possible under given conditions.

3. The choice of the maximum possible feed, taking into account technological and design constraints for each tool individually, and then for the tool spindle.

4. Determination of cutting speeds that provide the most advantageous tool life period corresponding to the minimum cost price of processing for the entire tooling.

The calculation of the tool life period is carried out depending on the variety of the tooling and is equal: a) their sum for the tools of the same name and equally loaded; b) the reduced amount, taking into account the ratio of the periods of the durability of each tool, working independently and in conditions of multi-tool adjustment. Also, the cutting time coefficient characterizing the share of each tool is taken into account.

Another feature is the concept of a limiting tool (for the least tool life in adjustment) and the appointment of the most advantageous cutting speed for this tool. The multi-tooling coefficient is introduced into the formula of limiting durability T_{lim}, taking into account the number of tools in adjustment and the uniformity of their loading. The NIBTN standards give numerical values of this coefficient. So for multi-spindle automatic machines take T_{lim} = 150 min. For the rational use of multi-spindle modular-type machines, it is necessary to equalize the running time of all carriage and positions by increasing the manufacturing pass time by correspondingly reducing feeds and spindle speeds.

For multi-tool adjustments, a large number of parameters affecting the machining process are characteristic. An analytical solution to the problem of optimizing cutting conditions reduces to determining the conditions for the maximum or minimum functions of many variables that are interconnected by many constraints dependencies. In the work of G.I. Temchin [20], the calculation of multi-tool adjustments based on finding the extremum of the many variables function the Lagrange multiplier method was considered. At the same time, there are several constraints on the number of optimized variables, for example, for automatic transfer lines where a large number of tools work, the number of decision variables can reach several hundred or more. Besides, the Lagrange method assumes the existence of derivatives of the optimized function at the point where the extremum is reached, and when optimizing the cutting conditions, some functions reach an extremum at the boundaries of the regions, i.e. where partial derivatives do not exist. Another necessary condition for the applicability of the Lagrange multiplier method is related to the equality of the number of equations and the number of unknowns; otherwise, the use of the substitution (exception) method for a large number of equations is practically not applicable.

Non-linear programming methods in the problems of optimizing operating modes on metal-cutting machines with multi-tool adjustments were used by A.M. Gilman [21]. The author reduces the problem of optimization according to the criteria of minimum cost price and minimum time per piece to finding such values of controlled variables that would ensure the minimum values of these criteria when fulfilling the given constraints and solving this problem by non-linear programming methods.

Along with productivity and cost price, reliability criteria are used as criteria for optimizing finishing processing – the average cutting time between T_{av} sub-adjustments and the machine utilization factor η_0, which, in turn, is associated with the specific wear parameter u_0 [22]. These criteria reflect the features of finishing and establish a relationship between the tool's workability and productivity with cutting speed. The author [19] obtained graphs of the specific wear u_0 and coefficient η_0 versus cutting speed. Using these graphs, the optimum cutting speed is obtained: Vo = 200 m/min (the machined material is 41Cr4 (steel 40X) – cutting tool inserts material of DIN HT01 (T15K6)) at which the specific wear will be optimal. It is at this speed (experiments show [19]) that the coefficient η_0 reaches its maximum and maximum productivity is achieved. Thus, based on the dependence $u_o = f\ (V)$, it is possible to determine the optimal mode of the highest finishing productivity and for the given cutting conditions to calculate the optimal spindle speed.

Features of the choice of optimal modes processing of parts on automatic transfer lines

In the works of G.I. Granovsky [23, 24] considered the problem of determining cutting conditions for automatic transfer lines, where the stable operation of all tools simultaneously working on the line is of great importance during the time between two adjacent moments of their group

replacement. The author experimentally proves that in this case, the calculation of cutting conditions, based on the selected tool life period, does not lead to optimal machining conditions. It is necessary to take into account such a factor as the number of parts K, which can be processed by the tool for its period of resistance. There is a dependence of the quantity K, which is directly proportional to the product $\{V{\cdot}T\}$. At the same time, tool life dependencies considered G.I. Granovsky describes a multi extreme curve, which is characteristic of the case of steel treatment with cemented carbide tool of the titanium-tungsten group and cemented oxide cutting tool. Based on the graphs $T = f\ (V)$ and $V{\cdot}T = f\ (V)$ shown in Fig. 1.1, a conclusion about the mismatch of the maxima of T and $V{\cdot}T$ is made. At the same time, the speed at which reaches a tool life maximum is less than the speed at which the $V{\cdot}T$ value maximum is reached, and therefore, maximum productivity. It was proposed to use a harmonic analysis of the Fourier series in [23] as a mathematical apparatus.

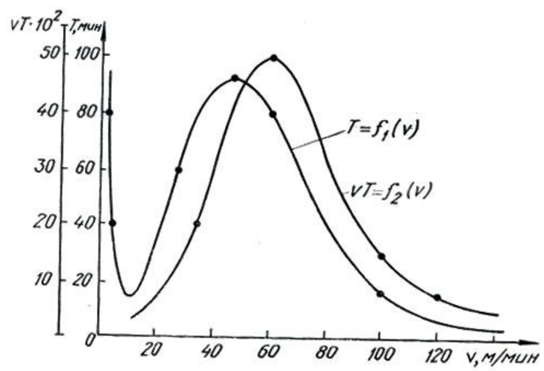

Fig. 1.1. The relationship between the dependences
$T = f\ (V)$ and $V{\cdot}T = f\ (V)$ [23]

Automated calculation of optimal cutting modes

Under the guidance of prof. Goransky G.K. at the beginning of the 1960s, work in the computer center of the Academy of Sciences of Belarus to automate the processes of determining the optimal cutting conditions on metal-cutting tools was carried out [8]. The linear programming method (MLP) is used as the main method for solving extreme problems. In favor of this method, the factor of ease the computer facilities using (computer technology of the 60s) worked when solving systems of linear equations and inequalities with which MLPs operate. This extremely simplified and accelerated the solution of even very complex and cumbersome tasks (for that time) under a large number of unknown and limiting conditions.

The advantage of MLP is the ability to operate with a large number of unknown optimized variables, which is typical for cases of automatic lines and automated equipment with multi-tool adjustment. Besides, as was already indicated [20], many functional dependences of the cutting process are characterized by an extremum at the boundary of function ranges, where the requirement for the existence of partial derivatives at extremum points is not fulfilled. For MLP, there are no constraints on the equality of the number of the equations on the one hand and the number of optimized variables on the other, which is typical, for example, of the Lagrange multiplier method. Linear programming encompasses methods for solving many optimal problems dealing with many interrelated variables subject to certain limiting conditions.

This all forms the field of rational use of the linear programming method.

The statement of linear programming problems can be formulated as follows: there is a certain parameter, for example, cost price, productivity, capital capacity, etc., which is a linear function of several variables. Variables, in turn, must satisfy the constraints expressed as a system of linear equalities or inequalities. It is required to find such non-

negative values of variables that satisfy the system of constraints under which the value, which is their linear function, takes the smallest or greatest value.

One of the main tasks of using MLP is to create a mathematical model that most fully describes the basic laws of the cutting process, which is then implemented on a computer. To compile a mathematical model, the equation of the objective function and the system of technical constraints are formed. As the objective function in the works of G.K. Goransky used the technological operation cost price or machining productivity. To calculate the effect of technical constraints on cutting conditions, they need to be expressed in the form of inequalities representing functions of the cutting modes elements. The main ten constraints are given in [8], which must be taken into account when working on metal-cutting machines. The mathematical model presented here is a description of the process of metal cutting, regardless of the type of machining, type of cutting tool, equipment and other processing conditions. For different types of machining, only the free terms in the inequalities of the constraints and the values of the coefficients in the equations describing the interconnections of the technical and economic parameters of the cutting process and the optimized variables will differ.

A mathematical model of the cutting process can be represented in graphical form, where the set of permissive cutting modes is a convex polygon, and the optimal cutting mode is achieved at one of the vertices of this polygon. This construction of the model for the optimization of cutting modes problem allows you to effectively carry out the automated calculation on a computer.

The organization of the implementation of the subsystem for calculating cutting modes and time norms was considered in [25]. For this, a systematic analysis of the unit's organizational structures was carried out, which was reduced to an operational description of these structures. As a rational option, the option of an industrial center for calculating cutting modes and time norms associated with a research laboratory for metal cutting was selected. The main tasks performed by the center were to unify the calculations for various cutting conditions, speed up the time and reduce the cost of technological preparation, develop calculation algorithms taking into account the specific conditions for machining parts at various enterprises. The decisive condition for the successful implementation of the parameters calculated by the industry center is the ability to obtain them for specific conditions for the parts manufacturing, depending on the combination “type of machining – the material being processed – cutting tool” without changing the software.

An application complex for calculating and optimizing cutting modes in the MathCAD and Exel software environments was presented in [26]. The database of cutting modes is formed in Exel, and the analytical and graphic parts (formulas, calculation results and graphs) are concentrated in MathCAD. So the problem of the optimal combination of factors of the cutting process for a given roughness is found by solving the system of equations in MathCAD. Similarly, the search for optimal modes that limit tool life and rigidity of the technological system is carried out.

In [27], a computer application was described that allows one to synthesize optimal variants of the cutting process on multioperational machines according to one of eight efficiency criteria, including two software modules:

The first module “Mode” – the module for calculating options for cutting modes;

The second module of modeling the machine operation when performing the technological process at the selected cutting modes by the method of automated construction of cyclograms.

In the program module “Mode” for all types of technological step (milling, drilling, boring, countersinking and reaming) by 30 variants of cutting modes are generated, differing in minute feed, tool life and

machine time. This makes it possible to select any variant of cutting modes to evaluate the effectiveness of its use in the CNC control program according to the current efficiency criteria.

The modeling algorithm [27] consists in the automatic synthesis of a chain of consecutive interdependent periods for a technological pass, as well as elementary components of the tool and workpiece change cycles. The initial data for the operation of the simulation program includes the technical characteristics of the machine, information about technological processes, including cutting modes, the nomenclature and tool life of instruments, as well as information about the initial placement of tools in the machine storage. The modeling program has two operating modes: simulation of the current dispatch list (DL) and modeling on its sample options (statistics).

In the simulation model of the current DL, a text file is created with a digital representation of the simulation results. Digital data are interpreted on a separate screen form by way of a cyclogram of the operation of the machine and its units, as well as in the form of a summary pie chart of the balance of time and indicators of the efficiency of the machine.

The analysis according to one or several of the criteria used for different machines, various cutting modes, options for placing tools in the nests of the storage can be carried out. The organization and use of the database on various models of multioperational machines available at the enterprise are possible. The purpose of the analysis can be the selection of the best machine according to the criteria used from the existing stock of machine tools, the formation of technical specifications for the purchase of a new machine, optimization of cutting modes, the stock of cutting tools, the planned duration of the machine cycle and so on.

Using the developed method [27] of formalized calculation and evaluation of cutting modes according to the current criteria for the performance of multioperational machines, a software product has been developed that allows to automatically generate up to 30 variants of cutting modes and evaluate their effectiveness according to eight criteria. The software product can be used at the enterprises of the machine-building profile, both for the operational synthesis of cutting modes according to the current production planning criteria and for reasonably formed technical specifications for the project requirements of new multioperational machines.

Integration of the software product with modern CAD systems used in industrial enterprises is possible by automatically substituting the operating parameters into the text of the NC control program, for example, in the STL, UNICODE formats; through exchange files, for example in the DBF, ADM, STEP formats; by dialog transfer to an adjacent graphic system window, for example, in CATIA, CIMATRON, PROENGINEER. Further expansion of the software product functionality is carried out in the direction of automatic optimization of cutting modes and their transfer to the control program of the CNC machine's system.

1.2. Statement the optimization problem as a task of optimal cutting conditions determination

In machine-building production, as in all other areas of human production, almost always there are problems of choosing the most effective solution to a multitude of technically (technologically) feasible ones. When designing the technological process of product manufacturing, these tasks take on specific forms related to the choice of machining methods, composition and sequence of technological operations, technological equipment, dimensional characteristics and cutting conditions.

Any technological process (operation, transition) must be efficient (optimal), i.e. best from the positions of the chosen optimality criterion K_0. At the same time, various constraints must be satisfied that form the

range of admissible solutions. For a specific version of the machining process, this thesis means the need to calculate optimal cutting modes, i.e. search for optimal values of the desired process parameters $\overline{X}$.

In addition to the required parameters, the technological process is characterized by a combination of phase parameters $\overline{Y}$, which are functions of the desired (independent) parameters (for example, cutting force and power, tool life, etc.). In addition, the mathematical model of the process includes the initial parameters $\overline{U}$: the coefficients of equations and other constants for the problem being solved [28].

All constraints are presented in the form of an inequalities system, each of which gives the admissible limits of variation for the corresponding phase or sought parameter (for example, the largest admissible value of cutting force, spindle speed, etc.). Analytically, this can be written as

$$X_{i\,\min} \le X_i \le X_{i\,\max}, \qquad Y_{j\,\min} \le Y_j \le Y_{j\,\max},$$
$$X_i \in \overline{X}, \quad Y_j \in \overline{Y}.$$

The optimality criterion is a function of the initial and required parameters:

$$K_0 = K_0(\overline{X}, \overline{U}).$$

It is assumed that this criterion must be either maximized ($K_0 \to \max$) or minimized ($K_0 \to \min$). The search for the vector of the required parameters that afford a maximum (minimum) value of the criterion K_0 is carried out on some set G of permissive solutions:

$$K_0(\overline{X}, \overline{U}) \xrightarrow[\overline{X} \in G]{} \max(\min)$$

Thus, the task of finding the optimal cutting modes is described by the following mathematical model:

$$K_0(\overline{X},\overline{U}) \xrightarrow[\overline{X}\in G]{} \max(\min);$$

$$y_j = \phi_j(\overline{X},\overline{U}), \quad y_j \in \overline{Y};$$

$$x_{i\min} \le x_i \le x_{i\max}, \quad x_j \in \overline{X};$$

$$y_{j\min} \le y_j \le y_{j\max}.$$

In the general case, there can be several optimality criteria (an analysis of such a situation will be made in Chapter 4).

The resulting model allows us to classify the initial optimization problem as a mathematical programming problem, which, depending on the type of constraints and the objective function, belongs to one of the following classes of problems: linear, nonlinear, discrete, dynamic and stochastic programming.

1.3. Optimality criteria for calculation of cutting modes

When searching for optimal cutting modes, the most varied criteria for choosing solutions can be used. Each of them reflects various aspects of the cutting process: economic, technical, physical, informational. The use of one or another criterion depends both on the objectively developing production situation and on the subjective views (experience, intuition) of decision-making engineers.

Most widely used in the practice of calculating optimal cutting modes are criteria for maximum productivity and minimum cost price. For machining conditions with one tool, the objective function that relates the cost price to the cutting conditions is:

$$C = a(t_c + t_{ch}\frac{t_m}{T}) + a'\frac{t_m}{T},$$

where C – the part of the technological cost price, depending on the cutting conditions; a – the cost of a machine-minute; t_c –time in cut; t_{ch} – tool change time; t_m – machine time of the workpiece cutting; T – tool life; a' – cost of the tool, reduced to one period of tool life (depends on the type of tool). For solder tool:

$$a' = \frac{\textit{Instrument cost + re-grinding costs}}{\textit{number of regrinds + 1}}.$$

For tools with multi-edged, non-grindable inserts:

$$a' = \frac{\textit{tool tip cost}}{\textit{number of cutting edge}} + \frac{\textit{tool holder cost}}{\textit{number of cutting edges per holder}}.$$

Processing productivity is inversely proportional to the time spent. Therefore, as part of the objective function, the part of the piece-production time calculation, depending on the cutting modes, is used:

$$t'_{pc} = t_c + t_{ch}\frac{t_m}{T}..$$

We study the function of the machining cost price. After trivial transformations, it can be represented as:

$$C = \frac{K_1}{Vf} + \frac{K_2}{VfT},$$

where K_1, K_2 are constant values.

It can be seen from this expression, that to reduce the cost price, it is necessary to strive, on the one hand, to increase the product $V \cdot f \to \max$, and on the other hand, to increase the tool life $T \to \max$ or $V^m f^n \to \min$. Since $m > 0$ and $n > 0$, a certain contradiction arises when trying to satisfy both requirements simultaneously. As a result, to simplify the search for optimal solutions, it is necessary to select a subset of admissible values of V and f, which in its properties would be similar to the Pareto set. In other words, if one goes from one point (solution) to another in this set, then one of the above criteria will improve, and the other will worsen.

Let the range of admissible values given by the inequalities:

$$V_{\min} \le V \le V_{\max}, \; f_{\min} \le f \le f_{\max}.$$

We construct on this set the function level lines $\varphi_1 = V \cdot f$ and the function $\varphi_2 = \mathrm{V^m\ S^n}$ (Fig. 1.2). Since $m > n$, the function level lines φ_2 will be "steeper" than the function level lines φ_1.

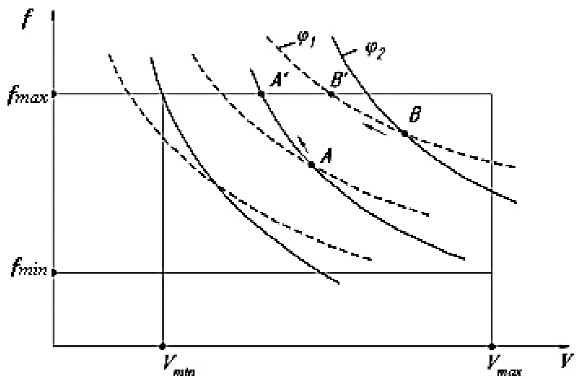

Fig. 1.2. The function lines level φ_1 and φ_2

Take an arbitrary point A (feasible solution). If you move along the line-up, you can see that the point A' is the best of the points lying on the line φ_2 since the value of the product $V \cdot f$ at this point is the greatest.

Let us take an arbitrary point B. If we move along the line-up, then we see that point B' – the best of the points lying on the line φ_1, since the value of the expression $V^m f^n$ at this point is the smallest. Thus, it can be argued that the set of pseudo-Pareto points lie on the lines $f = f_{\max}$ and $V = V_{\min}$.

If the Range of Permissible Values (RPV) is limited by a closed curve of arbitrary shape, then optimal solutions should be sought at the boundary of the RPV between the maximally scattered points of tangency of the lines φ_1 and φ_2 with the specified boundary (Fig. 1.3).

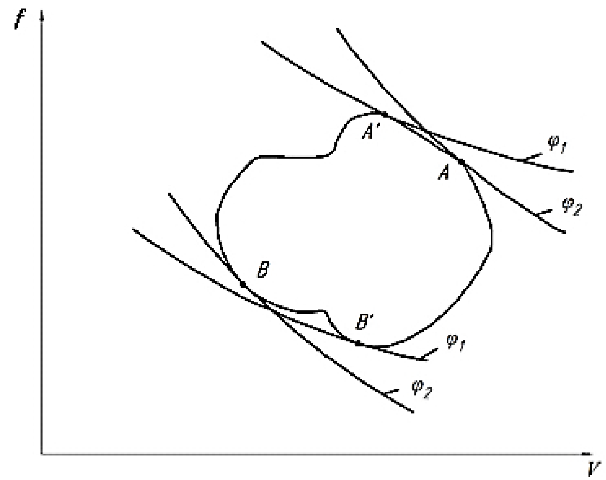

Fig. 1.3. Pseudo-Pareto value domain under arbitrary constraints

When moving from B' to A, the desired region in Fig. 1.3 is between points B' and A. Let $t_c = \frac{\pi D \ell}{1000\ V \cdot f}$ (where D – the diameter of the surface to be treated, l – length) and. Then the function C can be represented as follows:

$$C = \frac{a + bV^m f^n d^k}{V \cdot S}, \quad b = \frac{at_{ch} + a'}{k_t C_t}, \quad k = \frac{\pi D \ell}{1000}.$$

From the conditions of minimum cost price can be written $\frac{\partial C}{\partial V} = 0$ and $\frac{\partial C}{\partial f} = 0$.

Solving these equations, we obtain:

$$\begin{cases} \frac{bV^m f^n d^k (m-1) - a}{V^2 \cdot f} = 0; & (1.1) \\ \frac{bV^m f^n d^k (n-1) - a}{V \cdot f^2} = 0. & (1.2) \end{cases}$$

Since $m > n$, these equations cannot be valid simultaneously, which means the absence of a unique solution. For any feed value f, the cutting speed satisfying equation (1.1) will be less than the speed calculated from equation (1.2). The graphs of these equations are shown in fig. 1.4.

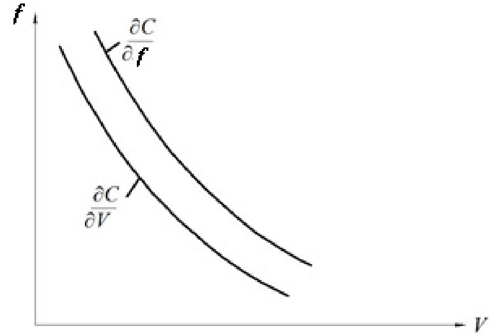

Fig. 1.4. Dependency graphs $\frac{\partial C}{\partial V} = 0$ and $\frac{\partial C}{\partial f} = 0$

To optimize cutting conditions, the criterion of maximum profit is often used. Profit per unit time:

$$P_t = \frac{D_{pp} - C_{pp}}{t_{pc}},$$

where D_{pp} – income per one part, excluding the cost of the material; C_{pp} – cost price of one part machining; t_{pc} – time per piece.

Profit earned per unit of funds spent P_c, income per one part, excluding cost:

$$P_c = \frac{D_{pp} - C_{pp}}{C_{pp} + Z},$$

where Z – additional cost.

It should be noted that in the general case, the cutting speed corresponding to the maximum profit per unit of time will differ from the cutting speeds corresponding to the minimum cost price and maximum productivity.

To find the optimal cutting conditions, the following criteria are also used:

1. The ratio of the volume of the removed material to the cost price C_{pr1} or time per piece C'_{pr1}:

$$C_{pr} = \frac{\pi D l d}{1000C} = \frac{VfT}{a + bV^m f^n d^k} \to \max .$$

2. The ratio of productivity to cost price C_{pr2}:

$$C_{pr2} = \frac{1}{t'_{pc}\ C} = \frac{V^2 f^2}{(1 + b'V^m f^n d^k)(a + bV^m f^n d^k)} \to \max,\ b' = \frac{t_c}{k_t C_t} .$$

3. The ratio of the amount of work done to the cost price C_{pr3}:

$$C_{pr3} = \frac{A}{C} = \frac{P_z L}{C} = k' \frac{V^{\alpha_1 + 1} f^{\alpha_2} d^{\alpha_3}}{a + bV^m f^n d^k} \to \max,$$

where $P_z = k_{pz} C_{pz} V^{\alpha_1} f^{\alpha_2} d^{\alpha_3}$, $k' = const$.

4. The ratio of the volume of material removed in one minute of work, to minute costs price C_{pr4}:

$$C_{pr4} = \frac{VfdT}{aT + a't_c} \to \max .$$

Each of these criteria has its advantages and disadvantages. However, one should point out their complex nature, due, in fact, to the multiplicative “convolution” of simpler criteria.

The accuracy of machining, tool life, product surface quality indicators (roughness, waviness, residual stresses, etc.), product durability indicators (wear resistance, endurance strength, contact

stiffness, etc.), operation reliability, machining stability, equipment efficiency can also be used as optimality criteria.

Of particular interest are information criteria. The analysis of information connections in the technological process allows a new approach from the cybernetic point of view to the consideration and synthesis of technology. From these positions, the primary forming process can be considered as a process of transferring information from a drawing of a part to a workpiece. Thus, it is correct to speak not only about energy and technology but also about the information productivity of the process. In a first approximation, the mass of information I contained in the structure of the machined part:

$$I = S\ (1 + \sum_1^n \frac{R_i}{\Delta R_i} \log_2 \frac{R_i}{\Delta R_i}) ,$$

where S – surface area; n – number of sizes obtained; R_i – i-th received size; ΔR_i – the accuracy of the i-th size.

1.4. Technical constraints in the optimization task of cutting modes

Correct calculation of optimal cutting conditions is not possible without taking into account constraints. Consider some of the constraints types that are most widely used in the theory and practice of mechanical engineering.

1. Kinematic constraints determine the maximum and minimum possible feeds and cutting speeds and are characterized by the kinematic structure of the main drive and feed drive.

2. Constraints on the maximum power of the machine. For cutting it is necessary that $N_c \le N_m \eta$, where N_c – cutting power; N_m – the power of the main drive electric motor; η – efficiency of the drive of the main movement. For turning, kW:

$$N_{pc}\frac{P_z V}{60\cdot 1020}=\frac{k_{pz}C_{pz}V^{\alpha_1}f^{\alpha_2}d^{\alpha_3}\pi Dn}{61200\cdot 1000},$$

where n – rotation frequency, rpm

3. Maximum torque constraint. For the machine trouble-free operation, it is necessary that $M_{sp} \geq M_p$; where M_{sp} – permissible torque on the spindle, set based on the strength of the weak link of the gearbox; M_p – torque caused by cutting force:

$$M_p=\frac{P_z D}{2000}.$$

4. Strength constraint of the weak link of the machine feed mechanism. We write the following equation regarding feed force P_{af}:

$$P_{af}\geq P_x+\mu(P_z+P_y)= P_x+0,1(P_z+P_y)\ ,$$

where μ – coefficient of friction. Given that $P_y \approx 0,4P_z$; $P_x = 0,25P_z$ we get $P_{af} \geq 0,39P_z$.

5. The constraint on the tool holder's strength. If we consider the cutter as a beam, loaded at the end with concentrated forces P_z and P_x, then the condition of its strength can be represented as:

$$\frac{P_z L_h}{W_z\cdot 10^{-7}}+\frac{P_x L_h}{W_x\cdot 10^{-7}}\leq\sigma_b\ ;\ W_z=\frac{BH^2}{6};\ W_x=\frac{B^2H}{6}\ ,$$

where L_h – the tool holder overhang, mm; W – a moment of resistance of the cutter holder section, mm^3; σ_b – permissible bending stress for the holder material, Pa; B, H – width and height of the holder, mm.

6. The strength constraint of the carbide plate. The cutting force when turning with a carbide-tipped tool is limited by its strength:

$$P_z\leq 34c^{1,35}d^{0,77}\left(\frac{\sin 60^\circ}{\sin\varphi}\right)^{0,8},$$

where c – thickness of the plate, mm; φ – the main angle of the cutter in the plan, degrees.

7. Constraint on the rigidity of the cutting tool. Under the action of the cutting force, the tool is elastically deformed, which affects the accuracy of the machining. Neglecting the forces of P_y and P_x, the bending deflection f_b of the cutter (which must not exceed the permissible value f_{bp}) during turning can be determined by the formula:

$$f_\text{b}=\frac{P_z L_h^3}{3E\text{I}}\leq f_\text{bp},$$

where E – elastic modulus of the tool holder material, Pa; I – a polar moment of inertia of the tool holder section (for a rectangular section $I=\frac{BH^3}{12}$, for around one $I = 0,05D^4$), mm^4. For rough turning – $f_{bp} \approx 0,1$ mm, for finishing – $f_{bp} \approx 0.05...0.03$ mm.

8. The constraint on the detail rigidity. Resulting force P_{zy} causing deflection of the part:

$$P_{zy}=\sqrt{P_z^2+P_y^2}\approx 1,1P_z.$$

Maximum detail bending deflection:

$$f_b=\frac{P_{zy}l^3\mu_d}{K_з EI}\leq f_{bp},$$

where l – detail length; K_z – coefficient depending on the method of (when detail restraining in the centers K_z = 100, when detail restraining the chuck and the rear center K_z = 140, when cantilever fixing in the chuck K_z = 2.4); μ_d – dynamic coefficient equal to the ratio of the static stiffness of the detail to its dynamic stiffness. The values of the dynamic coefficient μ_d are given in the table. 1.1.

Table 1.1

The values of the dynamic coefficient μ_d

Machining in the centers			Machining in the chuck and center		
Finish	Semifinish	Roughing	Finish	Semifinish	Roughing
1.2	1.38	1.39	1.16	1.3	1.31

Permissible values of the detail deflection must not exceed 0.25 ... 0.5 tolerance on the size T_d. Approximate values of this indicator: with rough turning: 0.2 ... 0.4 mm, with half turning: ≈ 0.1 mm, with finishing – 0.2 T_d.

9. Constraint on the roughness of the processed surface. There are both theoretical and experimental dependencies for calculating the roughness parameters. In general, these equations have the form:

$$R_a(R_{\max}) = K_R V^{\beta_1} f^{\beta_2} d^{\beta_3},$$

where K_R – constant value for a given tool geometry and characteristics of the processed and cutting materials.

10. Constraint on cutting temperature. To provide the required characteristics of the surface layer, it is often necessary to impose a constraint on the maximum cutting temperature. The equations for calculating the cutting temperature are:

$$\theta = K_\theta V^{\gamma_1} f^{\gamma_2} d^{\gamma_3},$$

where K_θ – constant.

There are also theoretical dependences for calculating the cutting temperature θ, however, their use is difficult due to the need to have a sufficiently large number of characteristics of the machining and cutting materials.

11. Constraint on the torque allowed by the clumping chuck. To calculate the specified moment, theoretical models for various designs of the chuck are used. It should be remembered that the clamping force decreases by about 10000 *N* with an increase in the rotation frequency from zero to 1500 *rpm*. It is believed that the clamping force at the maximum speed of the chuck should not be less than one-third of the clamping force in a static state.

12. Constraint on the minimum allowable ratio of the thickness of the chip to its width. For small values of this ratio, the power load on the cutter increases sharply, which leads to increased wear of the tool.

13. Constraint on the spindle rotation frequency, associated with the appearance of vibrations in the machine tool-arbor-workpiece system, has almost no analytical dependencies.

These constraints are technical in nature. In addition to them, planning and economic constraints can be taken into account. For example, the maximum permissive time in cut t_c, the minimum permissive tool life, etc. The formation of the entire system of constraints is a rather laborious task and, therefore, in practical cases, often only some of them are taken into account. Effective accounting of all possible constraints is possible only within the framework of an automated system for calculating optimal cutting modes.

2. GEOMETRIC PROGRAMMING

The solution of many technical problems cannot be reduced to a model of linear programming due to the presence of nonlinear dependencies that describe the relationship of the desired variables. To solve the problems of the non-linear type, some rather complicated non-linear programming methods are used, including the Lagrange multiplier method. The wide distribution of the latter is hindered by a rather complicated software implementation on computers and high requirements for the mathematical formulation of the problem. Starting from the 60s, the method of geometric programming (MGP) has been increasingly used in technology and economics [15].

2.1. Features of the geometric method programming

Method of geometric programming arose in connection with an attempt to take into account the features of the engineering problem statement to search for optimal design solutions. First of all, it is necessary to note the basic requirement of MGP, which consists in the fact that all components of the optimization problem should be expressed quantitatively in the form of generalized positive polynomials *g(x)* called posynomials, from controlled parameters. In many technical applications, the objective function and constraints are presented as the sum of components $g(x) = U_1 + U_2 + ... + U_n$, each of which can be expressed as a power function

$$U_i = C_i X_1^{\alpha_{i1}} X_2^{\alpha_{i2}} ... X_m^{\alpha_{im}} \qquad (i = 1,...,n),\ (j = 1,...,m), \qquad (2.1)$$

where C_i – positive constant; α_{ij} – arbitrary real numbers; X_j – optimized parameters.

In contrast to the classical indirect methods (Lagrange multipliers, etc.), the extreme value of the objective function and the relative contribution of each component to its value is first found in the MGP, and then the optimal values of the variable parameters X_j^* are found.

In the problem research of minimizing posynomial (2.1), we use the well-known inequality

$$(U_1 + U_2)/2 > U_1^{1/2} U_2^{1/2}, \qquad (2.2)$$

where U_1, U_2 are non-negative numbers.

When considering geometric inequality with n non-negative numbers, when some of them are equal, the concept of weighted averages is introduced. In such a generalized form, geometrical inequality can be used to find lower-bound estimates for posynomials. The simplest statement of the GP problem does not consider compelled (active) constraints (inactive constraints include requirements for the sign of posynomial constants C_i and the sign of optimized variables X_j). In the general case, for arbitrary positive numbers $U_1, U_2, ..., U_n$ and such positive weights $w_1, ..., w_n$, for which $\sum_{i=1}^{n} w_i = 1$, there is a relation $\sum_i w_i U_i \geq \Pi_i U_i^{w_i}$. Believing $w_i U_i = u_i > 0$, the last expression can be written as

$$\sum_{i=1}^{n} u_i \geq \Pi\left(\frac{u_i}{w_i}\right)^{w_i} = \left[\Pi\left(\frac{C_i}{w_i}\right)^{w_i}\right]\left[\prod_j X_j^{\sum_i \alpha_{ij} w_i}\right]. \qquad (2.3)$$

In inequality (2.3), the left-hand side is called the direct function *g (x)*, and the right-hand side is called the pre-dual function *V (w, X)*. The exponents for X_j are linear combinations $D_j = \sum \alpha_{ij} w_i$. If we choose weights w_i so that everyone D_j vanishes (tends to zero), then the pre-dual function will not depend on the variables X_j and will take the form $V(w) = \underset{i}{\Pi}(C_i / w_i)^{wi}$, then inequality (2.3) reduces to the system of relations:

$$g(x) = \sum u_i \geq \underset{i}{\Pi}(C_i / w_i)^{w_i}; \quad w_i > 0; \qquad (2.4)$$

$$\sum_i w_i = 1; \qquad (2.5)$$

$$\sum_i \alpha_{ij} w_i = 0. \qquad (2.6)$$

The system of relations (2.4) – (2.6) is called the dual MGP statement, according to which the problem reduces to maximizing the dual function *V (w)* (it was shown in [15] that the maximum of the dual function is equal to the minimum of the direct function *g (x)*) for linear constraints on dual variables called normalization conditions (2.5) and orthogonality conditions (2.6). Since inequality (2.3) implies that *g (x)* has a positive lower bound (a lower bound for posynomials), we can write down:

$$g(x) \geq M \geq V(w). \qquad (2.7)$$

From (2.7) it can be seen that *M* is an upper bound for the dual function for any choice of weights *w* for which the indices D_j vanish.

It should be noted that since equality (2.3) is achieved $w_i = u_i / \sum u$, when there is a simple connection between lines *X* and dual *w* variables.

Indeed, the optimal values of the direct variables X_j^* and the duals w_i^* satisfy the following relation:

$$w_i^* = C_i \underset{j}{\Pi}\left(X_j^{*\alpha_{ij}} / q\left(X^*\right)\right). \qquad (2.8)$$

Based on the foregoing, it is advisable to formulate the optimization problem for MGP in this sequence.

The direct task of GP:

Minimize $g(x) = \sum C_i \underset{j}{\Pi} X_j^{\alpha_{ij}}$

under constraints $X_j > 0; C_i > 0$.

The dual-task of GP:

Maximize $V(w) = \underset{i}{\Pi}(C_i / w_i)^{w_i}$

under constraints $\sum_i w_i = 1;\ \sum_i \alpha_{ij} w_i = 0;\ w_i \geq 0$.

In most real problems, the required variables are subject to active (compelled) constraints that do not follow from the statement given earlier.

The statement of the direct GP problem with constraints is presented below:

Minimize:

$$g_0(x) = \sum_{i_0=1}^{n_0} C_{0i_0} \cdot \Pi X_j^{\alpha_{0i_0 j}} \qquad (2.9)$$

under constraints: $X_j > 0,\ k = 1,2...,K;\ C_{ki} > 0;$

$$g_k(x) = \sum_{k=1}^{n_k} C_{ki_k} \cdot \Pi_j X_j^{\alpha_{ki_k j}} \le 1.$$

Here, the indices *0* and *k* relate, respectively, to the objective function and the constraints, which are posynomials containing n_0 and n_k $(k = 1,...,K)$, respectively, terms.

The dual problem is formulated in the same way as for the case of GP without constraints:

Maximize:

$$V(w) = \left[\Pi_i \left(\frac{C_{0i_0}}{w_{0i_0}} \right)^{w_{0i_0}} \right] \left[\Pi_k \Pi_{ik} \left(\frac{C_{ki_k}}{w_{ki_k}} \right)^{w_{ki_k}} \Pi_k \lambda_k^{\lambda_k} \right] \qquad (2.10)$$

under constraints:

$$\sum w_{0i_0} = 1\ (i_0 = 1,...,n_0); \qquad (2.11)$$

$$\sum_{i_0} \alpha_{0i_0 j} w_{0i_0} + \sum_{k=1}^{K} \sum_{i_k=1}^{K} \alpha_{ki_k j} w_{ki_k} = 0;$$

$$\lambda_k = \sum_k w_{ki_k};\quad w_{ki_k} \ge 0;\quad w_{0i_0} \ge 0;\quad \lambda_k \ge 0. \qquad (2.12)$$

In the above statement, the normalization condition is extended to weights w_{0i} characterizing the contribution of the members of the objective function. Weights w_{ki_k} are generally not normalized. We denote $\Delta_1, \Delta_2,..., \Delta_H$ the non-normalized weights, and λ – their sum: $\lambda = \Delta_1 + ... + \Delta_H$. Then the ratio between the normalized and non-normalized weights will take the form $\Delta\eta = \lambda W_h (h = 1,...,H)$. Replacing w_i by Δ_i / λ in the geometric inequality (2.3), we obtain:

$$\sum u_i \ge \Pi_i (u_i \lambda_i / \Delta_i)^{\Delta_i / \lambda} = \Pi_i \left(\frac{U_i}{\Delta_i} \right)^{\Delta_i / \lambda} \lambda.$$

The equivalent form of notation is inequality:

$$(\sum_i u_i)^{\lambda} \ge \Pi_i (U_i / \Delta_i)^{\Delta_i} \lambda^{\lambda}. \qquad (2.13)$$

The initial formulation of the direct GP problem with constraints (2.9), taking into account the above, can be represented as a minimization problem $g_0^{\lambda}(x)$ under constraints $g_k^{\lambda}(x) \le 1$. Then from geometric inequality (2.3), we have:

$$g_0^{\lambda_0}(x) \ge \Pi_{i_0} \left(\frac{U_{0i_0}}{\Delta_{0i_0}} \right)^{\Delta_{0i_0}} \lambda_0^{\lambda_0};\quad (i_0 = 1,...,n_0); \qquad (2.14)$$

$$1 \ge g_k^{\lambda_k}(x) \ge \left[\Pi_k \Pi_{i_k} \frac{U_{ki_k}}{\Delta_{ki_k}} \right]^{\Delta_{ki_k}} \left[\Pi_k \lambda_k^{\lambda_k} \right];\ (i_k = 1,...,n_k). \qquad (2.15)$$

Multiplying the geometric inequalities (2.14) and (2.15), we obtain:

$$g_0^{\lambda}(x) \geq \left[\underset{i_0}{\Pi} \left(\frac{U_{0i_0}}{\Delta_{0i_0}} \right)^{\Delta_{0i_0}} \lambda_0^{\lambda_0} \right] \left[\underset{k}{\Pi} \underset{i_k}{\Pi} \left(\frac{U_{ki_k}}{\Delta_{ki_k}} \right) \right] \left[\underset{k}{\Pi} \lambda_k^{\lambda_k} \right]. \tag{2.16}$$

This inequality holds for any choice λ, including for $\lambda_0 = \sum_i w_{0i} = 1$. We denote the weights normalized in this way, then relation (2.16) takes the form:

$$g_0(x) \geq \left[\underset{i_0}{\Pi} \left(\frac{U_{0i_0}}{w_{0i_0}} \right)^{w_{0\,i_0}} \right] \left[\underset{k}{\Pi} \underset{i_k}{\Pi} \left(\frac{U_{ki_k}}{w_{ki_k}} \right)^{w_{k\,i_k}} \right] \left[\underset{k}{\Pi} \lambda_k^{\lambda_k} \right]. \tag{2.17}$$

After obtaining the pre-dual $V(w, X)$ (right-hand side of expression (2.14)) and the dual functions $V(w)$ following (2.3), the formulation of the dual GP problem with active constraints reduces to (2.10) – (2.12). Moreover, constraints (2.11) and (2.12) are called normalization and orthogonality conditions respectively. The determination of the optimal values of direct X_j and dual w_i variables for the GP problem with active constraints is carried out similarly to (2.18):

$$w_{ki_k} / \lambda_k = C_{ki_k} \underset{j}{\Pi} \left(X_j^* \right)^{\alpha_{k\,i_k\,j}}. \tag{2.18}$$

When researching various GP statements, one should consider the case widespread in the theory of cutting, when both the objective function and the constraints are represented by monomial posynomial. Then for $k = 1, 2, ..., K \rightarrow \lambda_0 = w_0$ and $\lambda_k = W_k$.

In this case, the dual-task is formed as follows:

Maximize

$$V(w) = \left[\underset{i_o}{\Pi} C_{oi_0}^{w_{0i_0}} \right] \left[\underset{k}{\Pi} \underset{i_k}{\Pi} C_{ki_k}^{w_{ki_k}} \right]$$

under constraints $\sum_{i_0} w_{0i_0} = 1$ (confluent condition of orthogonality at $n_0 = 1$); $\sum_i \alpha_{ij} w_i = 0$ (orthogonality conditions formed for each unknown X_j $(j = 1, ..., m)$). Here n is the total number of objective function posynomials and all constraints.

When solving GP problems indirect and dual formulations, first of all, attention should be paid to the degree of difficulty, which is understood as the difference between the total number of terms n in the objective function and the constraints and the number of dual constraints $m+1$, where m is the number of optimized parameters. The problems with the zero degrees of difficulty are most computationally efficient [16].

If n is only one more than m, then the normalization and orthogonality conditions give a unique, easily obtained optimal solution for dual variables (weights) w^*. With an increase in the degree of difficulty (greater than zero), the corresponding system of linear equations does not has a unique solution and leads to variable weights, while there is no need to solve the dual problem as an optimization one.

There are several simple and effective techniques [29] for reducing GP problems to a zero degree of difficulty. The simplest of them is the method of unification, which consists of combining some members of the objective function based on their weights. In this case, the unification is performed before calculating the extremum of the objective function. It should be noted that only those members whose degree indicators differ slightly from each other should be combined. At the same time, the unification of members in which exponents with the same variables have opposite signs is avoided. For complex GP problems with constraints, where the unification may not be obvious, a technique called partial invariance is applied. By this approach, certain terms, the number of which is one more than the number of optimized parameters *m*, are accepted as dominant (these terms should be the most important in the optimal design). Next, a system of linear equations of the dual GP problem is solved concerning the dominant dual variables (i.e., variables corresponding to the dominant terms), which can be expressed in terms of other variables called basic ones w_i^b (the number of basic variables is equal to the degree of difficulty).

Based on the requirements for non-negativity w_i^b and taking into account the main weights of these variables (for the problem under consideration, these are the values of the objective function components, divided by their sum, calculated for a similar problem of zero difficulties), weights of the basic variables are selected. Based on the resulting system of dual dominant basic variables, the extreme value of the objective function is determined. If the obtained solution is unacceptable, it is necessary to implement an iterative procedure the searching for a system of dual variables by replacing the basic ones w_i^b. At the same time, it must be remembered that the above methods of reducing the GP problem to the zero degrees of difficulty are not applicable if there is no initial solution necessary for estimating dual variables.

2.2. Method of geometrical programming for zero degrees of difficulty

When optimizing the cutting modes, the most often used criterion of optimality is productivity (by machine time t_c) or the main time spent per unit length of cutting:

$$t_c = \frac{L\Delta}{fnd},$$

where L – length of the treatment, mm; Δ – allowance, mm; f – feed, mm/rev; n – rotation speed, m/s; d – the depth of cut, mm. Consider the procedure for forming a set of constraints. The most important technical constraints for many finishing methods are associated [8] with the cutting capabilities of the tool:

$$V \le V_T \to nf^{y_v} d^{x_v} \le \frac{1000C_v}{T^m \pi D},$$

where V_T – permissible speed value due to a given tool life period;

C_v, y_v, x_v – constant and degree indicators characterizing the influence of f and d on the cutting speed;

T – tool life period, min;

D – diameter of the part, mm.

The second constraints are associated with the requirements for the roughness of the treated surface:

$$R_z \le R_z^p \to f^{y_R} d^{x_R} \le \frac{R_z^p C_r r^{z_R}}{(\varphi \cdot \varphi_1)^{q_R}},$$

where R_z^p – permissive roughness value; φ, φ_1 – main and auxiliary angles in the plan; r – radius of rounding of the cutting blades;

The third constraints are related to the requirements for the maximum value of the depth of cut:

$$d \le \Delta.$$

To find the optimal values of the controlled variables $\{n_0, f_0, d_0\}$ that provide a minimum of machine time and satisfy the above constraints, we use the GP method of zero degrees of difficulty (T-N-1 = 4-3-1 = 0) [15]. In many practical problems, the direct formulation of the GP method reduces to the dual formulation [16, 29], which consists in maximizing the dual function

$$V(\delta) = \left(\frac{C_1}{\delta_1}\right)^{\delta_1} \cdot \left(\frac{C_1}{\delta_2}\right)^{\delta_2} \times \ldots \times \left(\frac{C_n}{\delta_n}\right)^{\delta_n}.$$

for a given system of constraints – linear equations:
- condition of orthogonality:

$$\sum_{i=1}^{n} \delta_i \alpha_{it} = 0,$$

where δ_i – arbitrary positive weights satisfying (for the terms of the objective function):
- normalization condition

$$\sum_{t=1}^{T} \delta_{0t} = 1.$$

For a special case, when the objective function and constraints are one-term posynomials, the normalization conditions are written in form $\delta_{01} = 1$, and the dual function $V_1(\delta)$ is maximized

$$V_1(\delta) = \left(\frac{C_{01}}{\delta_{01}}\right)^{\delta_{01}} \cdot C_{11}^{\delta_{11}} \cdot C_{21}^{\delta_{21}} \cdot \ldots \cdot C_{n1}^{\delta_{n1}} \to \max.$$

Implementation of the GP method of zero degrees of difficulty

Example 1. Find the optimal modes of finishing turning the shaft (the processing circuit is shown in fig. 2.1), providing a minimum cost price C [30, 31].

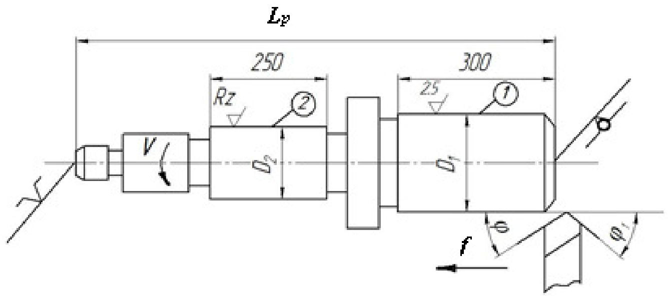

Fig. 2.1. Scheme machining of shaft

Initial data: screw-cutting lathe with CNC model 16K20F3; workpiece – rolled C45 (steel 45, σ_B = 598 MPa), D_2 = 100 mm, R_a = 2.5 μm, L_p = 860 mm, L_2 = 250 mm, allowance Δ = 1 mm; tool – a straight turning cutter tool with mechanical fastening of a hexagonal tip of cemented-carbide T15K6, φ = 45°, φ_1 = 10°; r = 1.0 mm, $\gamma = 10^0$.

The formation of the objective function. As an optimization criterion, we take part in the machining cost price with a cutting tool in one pass, which depends on the value of the cutting speed, feed and cutting depth. An analytical description of this relationship:

$$C = At_c + At_{ch} t_c / T, \tag{2.19}$$

where A is the total cost price of 1 min of the machine operation and the machine tool operator, cent/min; t_{ch} – tool change time, min.

To solve this problem (taking into account the ratio: $n = 1000V / \pi D$), we use the analytical expression:

$$t_c = \frac{\pi D L \Delta}{1000Vfd}. \tag{2.20}$$

The tool life equation for the turning operation is as follows:

$$T = C_T / V^{\frac{1}{m}} f^{\frac{y_v}{m}} d^{\frac{x_v}{m}}, \tag{2.21}$$

where C_T – coefficient takes into account the operating conditions of the tool; y_v, x_v, m – exponents determined from the known tool life equation [30].

After substituting (2.21) and (2.20) into equation (2.19), we obtain:

$$C = A\frac{\pi D L \Delta}{1000Vfd} + At_{ch}\frac{\pi D L \Delta}{1000C_T} V^{\frac{1}{m}-1} f^{\frac{y_v}{m}-1} d^{\frac{x_v}{m}-1}. \tag{2.22}$$

Formation of a system of technical constraints. For finishing, the constraint on the accuracy characteristics of the part (tolerance field, the accuracy of the surface shape and their relative position) is important. In addition to the accuracy constraint, we also impose a constraint on the maximum value of the cut depth. These two constraints should be explicitly stated.

Constraint 1. $P_y \le [P_y]$, where P_y, $[P_y]$ – radial component of the cutting force and its maximum permissible value. For explicit representation, it is necessary to use the relation $[P_y] = 3 f_p E \pi D^4 \mu / 64 L_p^3$ and the analytical transformation, as a result of which we obtain:

$$V^{n_p} f^{y_p} d^{x_p} \le \frac{3\mu_r f_{bp} E \pi D^4}{64 C_p K_p L_p^3}, \tag{2.23}$$

where x_p, y_p, n_p – the degree indicators characterizing the influence, respectively, of the cutting depth, feed and cutting speed on the value; P_y; μ_r – coefficient taking into account the peculiarities of the part restraining (for the mounting option in the chuck and center μ_r = 5.4); f_{bp} – permissive bending of the part (approximately 20% of the allowance) and for $R_z = 10 \to f_{bp} = 0{,}05$ mm; C_p – coefficient taking into account the influence of working conditions adopted in the reference-book as a basis on the power P_y; K_p – coefficient taking into account the difference of specific working conditions from those given in the reference book.

Constraint 2. The maximum cutting depth must not exceed the machining allowance, i.e. $t \le \Delta$.

For the given initial data, taking into account example 1, the components of the dependence (2.23) according to [30] take the following values: $C_p = 243$; $K_{p_y} = 1$; $x_p = 0{,}9$; $y_p = 0{,}9$; $n_p = -0{,}3$.

Then the dependence (2.23) is presented as: $V^{-0,3} f^{0,6} d^{0,9} \le 0{,}17$, and the constraint on the depth of cut: $t \le 1$.

The formation of a mathematical model of the problem. The statement of the problem (2.9) should be used, and when forming the dual problem, it is necessary to take into account the features of the objective function (two-term posynomial).

A direct statement of MGP:

Minimize

$$g_0(V,f,d) = C = 46,63V^{-1}f^{-1}d^{-1} + 0,82\cdot 10^{-12}V_1^4 f_1^{1,25}d_1^{0,75}$$

under constraints

$$5,77V^{-0,3}f^{0,6}d^{0,9} \le 1; \quad d \le 1 .$$

It should be noted that this direct problem is a GP problem of zero degrees of difficulty, and each constraint contains a single posynomial term $\lambda_k = \Delta_k = w_k$ (for k = 1, 2).

The dual GP task with restrictions:

Maximize

$$V(w) = \left(\frac{C_{01}}{w_{01}}\right)^{w_{01}} \left(\frac{C_{02}}{w_{02}}\right)^{w_{02}} C_{11}^{w_{11}} C_{21}^{w_{21}} \qquad (2.24)$$

under constraints

$$w_{01} + w_{02} = 1;$$
$$V: -w_{01} + 4w_{02} - 0,3w_{11} = 0;$$
$$f: -w_{01} + 1,25w_{02} + 0,6w_{11} = 0;$$
$$d: -w_{01} + 0,75w_{02} + 0,9w_{11} + w_{21} = 0. \qquad (2.25)$$

1. Solve the system of linear equations (2.25). In this statement, the system has a unique solution: w_{01} = 0.76; w_{02} = 0.24; w_{11} = 0. 67; w_{21} = 0.24. The peculiarity of this option is the possibility, at the first stage of the solution, to evaluate the contribution of each component of the objective function to the total cost price *C* (2.22). The cost of the first component associated with machine processing is 76%, and the component associated with a tool change is 24%.

2. Calculate the extremum of the objective function *C* (2.24). To do this, we find the maximum of the dual function (2.26):

$$\max V(w) = (46,63/0,76)^{0,76} \cdot \left(0,82/(10^{12}\cdot 0,24)\right)^{0,24} \cdot 5,77^{0,67} \cdot 1^{0,21} = 0,13. \qquad (2.26)$$

3. Determine the optimal values of the elements of the cutting mode. Using relations (2.25) and (2.18), we compose a system of equations for determining the optimal cutting modes:

$$0.13\cdot 0.76 = 46.63\cdot V^{-1}f^{-1}d^{-1};$$
$$0.13\cdot 0.24 = 0.82\cdot 10^{-12}V^4 f^{1.25}d^{0.73};$$
$$1 = 5.77\cdot V^{-0.3}f^{0.6}d^{0.9};$$
$$1 = t.$$

Having solved this system of equations, we obtain the optimal values of the elements of the cutting modes:

V^*= 421.39 m/min; f^* = 1.12 mm/rev; t^* = 1 mm.

Example 2. Find the optimal modes of finishing turning the shaft by the criterion of minimizing the time in cut t_c [32].

We realize the formulation of the three-parameter problem of finding the optimal cutting modes using the example of machining a steel part C45 (Ct 45) (σ_B = 598 MPa; E = 2.1·10^5 MPa) with the longitudinal turning of a cylindrical surface with a diameter of D = 120 mm and a length L_2 = 250 mm. Using a tool (material of the cutting part DIN HS 123 (T15K6); φ = 45 °; φ_1 = 10 °; abrasive tool durability T = 60 min), it is necessary to remove the allowance Δ = 0.8 mm and obtains a surface roughness R_a = 10 µm.

Using the reference data [30], the mathematical model of the finish machining process for the case of the direct setting of GP can be represented as:

$$t_c = \frac{200}{fnd} \to \min \tag{2.26}$$

under constraints

$$\begin{aligned} &\frac{1}{613.45} f^{0,2} n d^{0,15} \le 1; \\ &\frac{1}{0.49} f^{0,7} d^{0,2} \le 1; \\ &\frac{1}{0.8} d \le 1. \end{aligned} \tag{2.27}$$

Since each constraint contains a single posynomial term, the problem dual to the considered GP problem is written as:

$$V_1(\delta) = \left(\frac{200}{\delta_{01}}\right)^{\delta_{01}} \cdot \left(\frac{1}{613,45}\right)^{\delta_{11}} \cdot \left(\frac{1}{0,49}\right)^{\delta_{21}} \cdot \left(\frac{1}{0,8}\right)^{\delta_{31}}$$

under linear constraints:

- normalization condition (degenerate): $\delta_{01} = 1$;
- conditions of orthogonality:

$$\begin{aligned} &-\delta_{01} + 0,2\delta_{11} + 0,7\delta_{21} = 0; \\ &-\delta_{01} + \delta_{11} = 0; \\ &-\delta_{01} + 0,15\delta_{11} + 0,2\delta_{21} + \delta_{31} = 0. \end{aligned} \tag{2.28}$$

Having solved the system of linear equations, we obtain the values of dual variables:

$$\delta_{01} = 1;\ \delta_{11} = 1;\ \delta_{21} = 1.14;\ \delta_{31} = 0.62. \tag{2.29}$$

We determine the optimal value of the objective function. According to the work [2]:

$$\min t_c = \max V_1(\delta).$$

Then, following the basic geometric inequality and using the results of solving the dual problem (2.28), we obtain:

$$t_c(n, f, d) \ge \frac{250 \cdot 0.8 \cdot 2.26 \cdot 1.25}{613.45} = 0.85 \text{ min}.$$

To determine the optimal values of the controlled variables f_0, n_0, d_0, when each constraint contains a single posynomial term, we compose a system of nonlinear equations [15]:

$$\begin{aligned} &0.85 = \frac{200}{fnd}; \\ &1 = \frac{1}{613.45} n f^{0.2} d^{0.15}; \\ &1 = \frac{1}{0.49} f^{0.7} d^{0.2}; \\ &1 = \frac{1}{0.8} d. \end{aligned}$$

Having solved this system, we obtain d_0 = 0.8 mm; f_0 = 0.38 mm/rev; n_0 = 762 min^{-1}; V_0 = 287.1 m/min.

When machining structural steel (σ_B = 560...620 MPa) with a cutting depth $d \leq 1.4$ mm, feeding $f \leq 0.38$ mm/rev with carbide tools (DIN HS 123 (T15K6), φ = 45^0 ... 60^0) with a cutting speed of V = 268 m/min, machine time decreases by 10 % in comparison with tabular methods for assigning cutting modes [1], which increases the productivity of finishing processing and allows you to more fully use the capabilities of the machine tool system.

The increases in the dimensionality of the constraints system (power, precision, etc.) due to the representation of the objective function $\frac{L\Delta}{fnd}$ as the sum of several posynomials:

$$\frac{L_1\Delta}{fnd} + \frac{L_2\Delta}{fnd} + \ldots + \frac{L_n\Delta}{fnd},$$

leads to the appearance of an incompatible system of linear equations in a dual formulation (the system has no solutions).

The GP method does not apply to the case when the exponents for unknowns in the objective function (2.26) and one of the constraints (2.27) are negative. In this case, the corresponding dual variable δ_{it} is negative, which contradicts the initial constraints in the GP, i.e. $\delta_{it} \geq 0$. This corresponds to the task of finding optimal cutting modes with a cemented-carbide tool (2.26), where the accuracy is considered as one of the constraints, in which the exponent for n is less than zero.

The GP method is not applicable if there are no constraints on at least one of the unknown parameters. This leads to a contradiction of the normalization and orthogonality conditions in the statement (2.28).

The use of the GP method will allow you to choose the optimal cutting conditions that provide increased processing productivity, more complete use of the capabilities of metal-cutting equipment and tools.

2.3. GP method of the 1st degree of difficulty

In the practice of technological design, optimization problems of large dimensions are often encountered, which cannot be solved by the method of geometric programming (GP) of zero difficulty degree [15; 16; 29], since they have at least one degree of difficulty. In this case, the dual GP problem is considered as an optimization one and reduces to searching for the extremum of the only redundant, dual variable values [15].

The use of the combined method of geometric and linear programming (with the procedure of linearizing nonlinear terms of a dual function by piecewise linear approximation) [33], as well as numerical of the reduced gradient method [16], are associated with the considerable complexity of computer software implementation. In work [34], three effective methods for numerically solving design problems are presented using the first-degree GP method, which reduces the overall complexity of the optimization calculations implementation.

Let us consider the use of the partial invariance method in problems of optimizing cutting modes on the example of a typical task of single-tool cutting. The implementation mechanism of this method will be reduced to bringing optimization problems to zero degrees of difficulty.

Machining scheme - finishing turning of the shaft in the centers; machine - screw-cutting lathe 16K20F3; stock – rolled stock, DIN C45 (steel 45, $\sigma_в$ = 598 MPa), diameter of the part machined surface D = 100 mm; microroughness height Ra = 2.5 μm; part length L_p = 860 mm; machining length L_2 = 250 mm; side allowance Δ = 0.7 mm; the tool is a lathe tool with mechanical fastening of a hexagonal tip made of DIN HS 123 (T15K6 hard alloy, φ = 45 °, φ_1 = 10 °, r = 1.0 mm, γ = 10 °).

The objective function is the cost price of machining:

$$C = A \cdot t_c + A \cdot t_{ch} \cdot t_c / T \ , \tag{2.30}$$

where A = 0.594 – the total cost of one minute of machine and machine tool operation, c/min; t_c, T – respectively, machine processing time and tool life, min; t_{ch} = 0.08 – tool change time, min.

Using the well-known [30] relations for t_c, and T, we represent the cost price:

$$C = \frac{A\pi DL\Delta}{1000 \cdot Vfd} + \frac{At_{ch}\pi DL\Delta}{1000 \cdot C_T} \cdot V^{\frac{1}{m}-1} f^{\frac{y_v}{m}-1} d^{\frac{x_v}{m}-1}, \tag{2.31}$$

where C_T – coefficient takes into account the operating conditions of the tool; y_v, x_v, m – exponents determined by standards [30].

We impose restrictions on the depth of cut:

$$t < 0.7 \text{ mm}; \tag{2.32}$$

cutting speed

$$\text{V} < 250 \text{ m/min}; \tag{2.33}$$

Machining quality surface

$$f^{0.7} d^{0.2} \leq 0.49. \tag{2.34}$$

The method of partial invariance consists of iteratively using the property of geometric inequality at a minimizing point to search for optimal (fairly close to optimal) values of the controlled variables for the design problem.

Implementation of the GP method of the first degree of difficulty

Consider the algorithm for implementing this method on the example of the above typical problem [35].

1. We solve the problem of zero difficulty degree without an additional term in the objective function, that is, we determine the lower bound of the objective function, the value of the dual variables w and the optimal variables x_n. Consider the problem of determining the optimal cutting modes:

$$x_n = \{f^*, n^*, d^*\},$$

ensuring a minimum cost price of the workpiece turning process (machining scheme and data are given above). The objective function in this step is the cost price of machining:

$$C = \frac{AL\Delta}{fnd},$$

and the inequalities (2.32) – (2.34) are constraints.

We write the model of the initial (direct) GP problem of zero degrees of difficulty: minimize

$$C = 104.13 f^{-1} n^{-1} d^{-1} \tag{2.35}$$

under restrictions:

$$\begin{aligned} &1.25 \cdot 10^{-3} n \leq 1; \\ &2.04 f^{0.7} d^{0.2} \leq 1; \\ &1.43\, d \leq 1. \end{aligned} \tag{2.36}$$

In this case, the model of the dual GP problem (each constraint contains a single posynominal term) can be written as follows:

Maximize

$$V(\delta) = 104.13^{w_{01}} \cdot \left(1.25 \cdot 10^{-3}\right)^{w_{11}} \cdot 2.04^{w_{21}} \cdot 1.43^{w_{31}}$$

under restrictions

$$w_{01} = 1;$$
$$n: -w_{01} + w_{11} = 0;$$
$$f: -w_{01} + 0.7w_{21} = 0;$$
$$d: -w_{01} + 0.2w_{21} + w_{31} = 0.$$

As a result of solving the GP problem in a dual statement, we obtain the values of the dual variables: $w_{01} = 1$; $w_{11} = 1$; $w_{21} = 1.43$; $w_{31} = 0.71$. The lower bound (exact) of the cost price

$$C = 104.13\ (1.25 \cdot 10^{-3}) \cdot 2.77 \cdot 1.29 = 0.47\text{cent},$$

and the optimal values of the cutting mode elements:

$$n^* = 796.18\ \text{min}^{-1};\ f^* = 0.4\ \text{mm/rev};\ d^* = 0.7\ \text{mm}.$$

2. We forming a model of the nonzero degree GP problem difficulty by introducing an additional term in the objective cost price function that is associated with the cost of the tool changing. It is important to use the additional term for finishing machining methods, when, with an increase in cutting speed and a decrease in machine time, the tool change time increases. Such a two-membered objective function takes the form:

$$C = 104.13 f^{-1} n^{-1} d^{-1} + 1.94 \cdot 10^{-3} n^{4} f^{0} d^{-0.25}, \qquad (2.37)$$

where the constraints are expressions (2.36).

3. We determine the lower bound of the objective function in the statement (2.35...2.36). In this case, one should use the optimal values of the variables (n^*, f^*, d^*) that are obtained as a result of solving the original problem of zero difficulty degree (in this case, without taking into account the term associated with the tool change). In this case, the machining cost price C^1 (cent):

$$C^1 = 104.13 \cdot 796.18^{-1} \cdot 0.4^{-1} \cdot 0.7^{-1} + 1.94 \cdot 10^{-13} \cdot 796.18^{4} \cdot 0.7^{-0.25} =$$
$$= 0.47 \cdot 0.085 = 0.555.$$

4. We calculate the basic weights of the objective function members by dividing the cost components by their sum. Moreover, the dual weights of the constraints $\{ w_{11},\ w_{21},\ w_{31} \}$ remain unchanged. A new set of dual weights has the form

$$w_{01}^{1} = 0.84;\ w_{02}^{1} = 0.16;\ w_{11}^{1} = 1.0;\ w_{21}^{1} = 1.43;\ w_{31}^{1} = 0.71. \qquad (2.38)$$

5. We compose a system of linear equations in the GP dual statement, including the conditions of orthogonality and normalization:

$$\begin{aligned} f:&\ \ -w_{01} + 0.7w_{21} = 0; \\ n:&\ \ -w_{01} + 4w_{02} + w_{11} = 0; \\ d:&\ \ -w_{01} - 0.25w_{02} + w_{31} = 0; \\ &\ \ w_{01} + w_{02} = 1. \end{aligned} \qquad (2.39)$$

6. Select the dominant members in the original statement. These should include the most significant members in the optimal project, and their number should exceed the number of project variables (f, n, d) by one unit. For this example, these members would be:

- first (($w_{01}^{1} = 0.84$);

- third $\left(w_{11}^{1}=1\right)$;

- fourth $\left(w_{21}^{1}=1.43\right)$;

- fifth $\left(w_{31}^{1}=0.71\right)$.

7. We solve the system of linear equations (2.39) for the dominant dual variables, ie variables corresponding to dominant members. In the case under consideration, linear dependences on w_{02}

$$w_{01}=1-w_{02};w_{11}=1-5w_{02};w_{21}=1.43-1.43w_{02};w_{31}=0.71-0.46w_{02} \quad (2.40)$$

For all values of w to be positive, w_{02} must be less than 0.2. As a first approximation, we take the main weight $w_{02}=0.16$. Then the dual variables

$$w_{01}^{2}=0.84;w_{02}^{2}=0.16;w_{11}^{2}=0.2;w_{21}^{2}=1.2;w_{31}^{2}=0.64. \quad (2.41)$$

8. We calculate the value of the lower bound of the objective function with new values of w_2 (2.41):

$$V(w^{2})=\left(\frac{104.13}{0.84}\right)^{0.84}\cdot\left(\frac{1.94\cdot10^{-13}}{0.16}\right)^{0.16}\cdot\left(1.25\cdot10^{-3}\right)^{0.2}\times \quad (2.42)$$
$$\times2.04^{1.2}\cdot1.43^{0.64}=0.55.$$

This value is less than the initial C^{I} with an accuracy of 1% (0.0055 cents) and cannot be considered as the first step of iterations. However, the partial invariance method can be used to more accurately approximate the optimum and search for the corresponding values of the variable cutting modes.

9. We determine the new values of the optimizing variables from the conditions of partial invariance. These conditions are called partial since the three optimizing parameters for the problem under consideration can be determined in several ways (5 conditions of invariance by the number of terms). In the process of solving, difficulties arise, firstly, with the need to check compatibility conditions, and secondly, with a multi-step analysis of various optimizing parameters. For the case under consideration, the optimizing parameters are determined from the following invariance conditions (taking into account the three dominant dual variables { w_{01}, w_{21}, w_{31} }):

$$\begin{aligned}&104.13f^{-1}n^{-1}d^{-1}=0.84\cdot0.55;\\&2.04f^{0.7}d^{0.2}=1;\\&1.43d=1.\end{aligned} \quad (2.43)$$

As a result of solving the system of equations (2.43), we obtain:

$$n^{*2}=804.96\text{ min}^{-1};\ f^{*2}=0.4\text{ mm/rev};\ d^{*2}=0.7\text{ mm}. \quad (2.44)$$

10. We determine the value of the objective function for a new combination of optimizing parameters (15):

$$C^{2}=\frac{104.3}{0.4\cdot804.96\cdot0.7}+\frac{1.94\cdot10^{-13}\cdot4.2\cdot10^{11}}{0.91}=0.46+0.0895=0.549.$$

The new value of the objective function is less than the cost of the initial version $C^{I}=0.555$, i.e., it gives the best design solution.

11. Choose the new dominant dual variables. At this stage, we choose the solutions as the dominant { w_{02}, w_{21}, w_{31} }. As a rule, when choosing a new set, one should take into account their ranking by

importance. Therefore, the new set should be in the form $\{w_{02}, w_{21}, w_{31}\}$. Considering the option when there is no dual variable w_{31}, i.e. a direct constraint on the depth of cut, which in turn leads to a violation of the integer conditions of the factor $\frac{\Delta}{d}$ in the first and second objective functions. So, using partial invariance for the set $\{w_{01}, w_{02}, w_{11}\}$ gives the following values of the optimizing parameters: n*= 796.18 min^{-1}; f* = 0.64 mm/rev; d* =0.14 mm. ($\frac{\Delta}{d}$ – not the integer)

For the set $\{w_{01}, w_{11}, w_{31}\}$, the partial invariance conditions are:

$$104.13 f^{-1} n^{-1} d^{-1} = 0.55 \cdot 0.84;$$
$$1.25 \cdot 10^{-3} n = 1;$$
$$1.43\, d = 1.$$

As a result of solving such a system of equations, we obtain a new design solution:

$$n^* = 796.18;\ d^* = 0.7;\ f^* = 0.41.$$

For this option, the cost is C = 0.5452 c., which is less than the initial C^1 and C^2 obtained as a result of the partial invariance procedure for a set of dual weights $\{w_{01}, w_{21}, w_{31}\}$.

Therefore, when turning the shaft (with the above initial data), the optimal machining cost is C = 0.5452. The share of costs for machine machining is 84%, for tool change – 16%, and the optimal cutting conditions are: n* = 796.18 min^{-1}; f* = 0.4 mm/rev; d* = 0.7 mm.

The software of GP the first-degree complexity is implemented in the mathematical environment of Maple.

The use of the GP method in problems of optimizing cutting modes of the first-degree difficulty is effective when using partial invariance

algorithms that provide a search for design solutions that are sufficiently close to optimal.

GP two-pass optimization

Consider an example of the implementation of MGP of the first degree of difficulty for two-pass turning. The task is formulated as follows: to find the optimal modes of two-pass turning (the machining scheme is shown in Fig. 2.1), ensuring a minimum technological cost price C [36].

Initial data: machine screw-cutting lathe 16K20F3; stock – rolled stock, basic material: DIN C70 (steel 70); D_1 = 100 mm, L = 260 mm, Ra = 2.5 μm; Δ = 5 mm; tool – lathe tool with mechanical fastening of a hexagonal tip made of H30 (T5K10 hard alloy); φ = 45; φ_1 = 10°; γ = 10; ρ = 1.0 mm.

The formation of the objective function. A variable part of the manufacturing cost of the part when turning in two passes can be written as:

$$C_2 = B\, t_{c1} + (B\, t_{ch} + B_T)\frac{t_{c1}}{T_1} + B\, t_{c2} + (B\, t_{ch} + B_T)\frac{t_{c2}}{T_2}, \qquad (2.45)$$

where B – the cost of machine time, cent/min; B_T – reduced costs arising from the operation of the cutting tool during its durability; t_{c1}, t_{c2} – cutting time, respectively, in the first and second passes; t_{ch} – tool change time; T_1, T_2 – a period of tool life, respectively, in the first and second pass.

For machining parts by turning, relations (2.20) and (2.21) are valid, taking into account which:

$$\begin{aligned} C_2 = {} & B\pi D_1 L/1000 V_1 f_1 + (Bt_{ch} + B_T) V_1^{\left(\frac{1}{m}-1\right)} f_1^{\left(\frac{y_1}{m}-1\right)} d^{\left(\frac{x}{m}\right)} / 1000 C_{T1} + \\ & + B\pi D_2 L/1000 V_2 f_2 + (Bt_{ch} + B_T) V_2^{\left(\frac{1}{m}-1\right)} f_2^{\left(\frac{y_1}{m}-1\right)} d^{\left(\frac{x}{m}\right)} / 1000 C_{T2}. \end{aligned} \qquad (2.46)$$

Formation of a technical system: constraints. The search for optimal cutting modes $\{V_{10}, f_{10}, V_{20}, f_{20}\}$ is carried out under active constraints on the cutting force for the first (rough) pass and the required surface roughness for the second (final) pass [37]:

$$C_p d^{x_P} f^{y_P} V^{n_P} K_p \le P_{pm}; \quad K_0 f^{k_1}(g_0+\gamma)^{k_H} / \rho^{k_2} V^{k_3} \le R_a, \tag{2.47}$$

where C_p, K_p, x_p, y_p, n_p – coefficients and exponents in the power dependence; P_{pm} – permissible cutting force; R_a – the roughness of the machined surface of the part shown in the drawing; ρ – radius at the cutter tip; γ – the front angle of the tool; k_0, k_1, k_2, k_3, k_4 – coefficients reflecting the influence of technological factors on R_a value

According to the reference data [37], the coefficients and exponents in expressions (2.43) take the following values: B = 2.43 c.; B_T = 13 c.; t_{ch} = 2 min; m = 0.2; y_1 = 0.45; y_2 = 0.35; x = 0.15; C_{T1} = 3.8·10^{11}; C_{T2} = 4.28·10^{11}; C_p = 339; x_p = 1; y_p = 0.5; n_p = -0.4; P_{pm} = 3600 H; k_0 = 5.8; k_1 = 1.1; k_2 = 0.68; k_3 = 0.15; k_4 = 0.45; d_1 = 4 mm; d_2 = 1 mm.

A direct statement of the optimization problem by the GP method is represented as

$$\begin{aligned} C_2 &= 190.85 V_1^{-1} f_1^{-1} + 0.99\cdot 10^{-8} V_1^4 f_1^{1.25} + \\ &+ 155.51 V_2^{-1} f_2^{-1} + 2.95\cdot 10^{-9} V_2^4 f_2^{0.75} \to \min \end{aligned} \tag{2.48}$$

under constraints:

$$3.77 V_1^{-0.4} f_1^{0.5} \le 1; \quad 28.38 V_2^{-0.15} f_2^{1.1} \le 1. \tag{2.49}$$

Since each constraint contains a unique polynomial term, the dual GP problem has the following form:

$$V(W) = \left(\frac{190.85}{w_{01}}\right)^{w_{01}} \left(\frac{0.99\cdot 10^{-8}}{w_{02}}\right)^{w_{02}} \left(\frac{155.51}{w_{03}}\right)^{w_{03}} \left(\frac{2.95\cdot 10^{-9}}{w_{04}}\right)^{w_{04}} .$$
$$\cdot 3.77^{w_{11}} \cdot 28.36^{w_{21}} \to \max$$

under constraints

$$\begin{aligned} V_1 &: -w_{01} + 4w_{02} - 0,4w_{11} = 0; \\ f_1 &: -w_{01} + 1.25w_{02} + 0.5w_{11} = 0; \\ V_2 &: -w_{03} + 4w_{04} - 0.15w_{21} = 0; \\ f_2 &: -w_{03} + 0.75w_{04} + 1.1w_{21} = 0; \\ & w_{01} + w_{02} + w_{03} + w_{04} = 1. \end{aligned} \tag{2.50}$$

Since the considered problem of the 1st degree of difficulty (m = 4; n = 6), the system of linear equations in the dual formulation does not have a unique solution. We use the Wilde method [34] to determine the dual weights w. To do this, we solve the system of linear equations (2.50) concerning the least significant tool component (the fourth posynomial of the objective function characterized by the dual weight w_{04}):

$$w_{01} = 0.73 - 3.37w_{04}; \qquad w_{02} = 0.27 - 1.24w_{04}; \qquad w_{03} = 3.61w_{04};$$
$$w_{11} = 0.81 - 4.02w_{04}; \quad w_{21} = 2.6w_{04}.$$

For everyone w to be positive (a necessary condition for GP), inequality $0 \le w_{04} \le 0.22$ must occur. We will represent *V(w)* as a function of w_{04} and, using the dichotomy method [38], we determine the exact upper boundary of the dual function $V(w_{04}) = 10.07$, the value of the dual weight maximizing the objective function $w_{04} = 0.165$, and after solving system (2.50), the values of the dual weights: $w_{01} = 0.19; w_{02} = 0.065;\ w_{03} = 0.6; w_{11} = 0.146;\ w_{21} = 0.43.$ We determine the values of the optimal cutting modes $\{V_{10}, f_{10}, V_{20}, f_{20}\}$ from the conditions of partial invariance [33]:

$$190.85V_1^{-1}f_1^{-1} = 10.07 \cdot 0.19;\ \ 3.77V_1^{0.4}f_1^{0.5} = 1;$$
$$155.51V_2^{-1}f_2^{-1} = 10.07 \cdot 0.6;\ \ 28.36V_2^{-0.15}f_2^{0.5} = 1. \qquad (2.51)$$

Having solved the system (2.51), we obtain the values of the optimal cutting conditions: V_{10} = 56.66 m/min; f_{10} = 1.76 mm/rev; V_{20} = 253.22 m/min; f_{20} = 0.102 mm/rev and the minimum cost price of two-pass processing C = 10.07 cent.

2.4. Express research procedure cost of two-pass processing

The traditional solution of the two-parameter optimization problem (in terms of cutting speed *V* and feed *f*) to search for optimal cutting conditions does not answer the question of the optimal cutting depth (*d*) and the distribution of the allowance for passages. For this, it is necessary to have an analytical dependence of the objective function (in this case, the machining cost price C) on the desired parameter *d* (at optimal cutting speeds V_0 and feed f_0). To construct such a dependence *C = f (d)* in the general case, it is necessary to implement the optimization procedure each time (by the number of calculated points), since a change in the cutting depth at the passage causes a change in the values of V_0 and f_0 [39]. In [36], the optimal variant of two-pass machining was obtained for some arbitrary partition of the allowance {d_1 = 4 mm; d_2 = 1 mm}.

For a reasonable purpose of the cutting depths, it is necessary to know the behavior of the cost price function of the two-pass machining C_2 on the cutting depth on the finishing pass, $C_2 = f(d_2)$. There are at least two alternatives: for this: construct this function from the

experimental points, solving the GP optimization problem for each combination of parameters $p_i = \{d_1; d_2\}$, or use the Wilde express method to construct the function $C_2 = f(d_2)$ [39], which significantly reduces the complexity of the calculations. The express methodology uses the constancy property of the polynomial terms relationship in the geometric programming problem. The parameters changing that determine the constraints imposed directly leads to the calculation of the lower boundary of the new minimizing objective function C_2^e. Using this property, the lower bound *y* for new parameter values p_i' is determined from the inequality [39]:

$$y \geq \sum_{j=1}^{R} w_j \sum_{i=1}^{q} b_{ji} r_i, \qquad (2.52)$$

where b_{ji} – exponents with the parameter p_i; $i = 1,...,q$ – the number of parameters.

In the matrix notation (2.52) represents [1]:

$$y \geq \vec{\boldsymbol{\delta}}^T \mathbf{B} \vec{r}, \qquad (2.53)$$

where $\vec{\mathbf{r}}$ – vector of dimension *q* from the variables *r*; $\vec{\boldsymbol{\delta}}^T$ – transposed vector of dual variables by dimensionality *R*; **B** – matrix of dimension $R \times q$.

The optimal values of variables $z = \{V_{10}; f_{10}; V_{20}; f_{20}\}$, or in exponential form $u_i = \ln\left(\frac{z_i'}{z_i}\right)$, are determined from the invariance condition in matrix form:

$$\mathbf{A}^T \vec{u} = \mathbf{M}\, \vec{r}, \qquad (2.54)$$

where $\mathbf{A}^T$ – transposed matrix of exponents for variables z; **M**-matrix, the elements of which are defined as follows:

$$m_{ji} = \begin{cases} g_j - b_{ji} & \text{при} \quad j \in J_0 \\ -b_{ji} & \text{при} \quad j \notin J_0 \end{cases},$$

here $g = \mathbf{B}^T \vec{\boldsymbol{\delta}}$ – set of indices with which the posynomials of the objective function are associated.

Transformation of condition (2.54) allows us to present the solution u in the form of a linear function $\vec{r}$ [1]

$$u = \left(\mathbf{A}\Delta^T \Delta \mathbf{A}^T\right)^{-1} \mathbf{A}\Delta^T \Delta \mathbf{M}\vec{\mathbf{r}} = \mathbf{P}\,\vec{\mathbf{r}}\,. \tag{2.55}$$

In the problem of optimizing the cost price of two-pass machining C_2 in the expression of the objective function C_2, there are two parameters $p_2 = \{d_1;\ d_2\}$ in three posynomial terms associated with the cost components of the tool change [40]:

$$0.35 \cdot 10^{-8} V_1^4 f_1^{1.25} d_1^{075};$$
$$2.95 \cdot 10^{-9} V_2^4 f_2^{1.25} d_2^{0.75},$$

and a constraint on the permissible axial cutting force $0.94\, V_1^{-0,4} f_1^{0,5} \le 1$. For the basic distribution of cutting depths $\{d_1 = 4$ mm; $d_2 = 1$ mm$\}$ and the corresponding optimal values of the cutting and feed rates, the coefficients C_{ij} as functions of d can be written in the following form:

$$C_{02} = 0.35 \cdot 10^{-8} d_1^{0.75}; \quad C_{04} = 0.295 \cdot 10^{-8} d_2^{0.75}; \;\; C_{11} = 0.94 d_1^1.$$

The matrix **B** and the vector $\bar{\boldsymbol{\delta}}$ for this case are

$$\mathbf{B} = \begin{matrix} C_{02} \\ C_{01} \\ C_{11} \end{matrix} \begin{bmatrix} 0.75 & 0 \\ 0 & 0.75 \\ 1 & 0 \end{bmatrix}; \quad \vec{\boldsymbol{\delta}} = \begin{pmatrix} 0.06 & 0.16 & 0.14 \end{pmatrix}.$$

Using analytical dependences for machine time and tool life [30] and specific input data [40] for pricing $\mathbf{B}^T \vec{\boldsymbol{\delta}}$, after potentiation we define the following cost boundary as a power-law function of the parameter ratio q_i:

$$C_o' \ge C_0 q_1^{0.195} q_2^{0.124}, \tag{2.56}$$

where C_0', C_0 – new and basic (in this example, for parameter values d_1 = 4; d_2 = 1) values of the objective function. The optimal values of the variable cutting conditions are determined from (2.55) using the operations of matrix algebra

$$\mathbf{A} = \begin{bmatrix} -1 & 4 & 0 & 0 & -0.4 & 0 \\ -1 & 1.25 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & -1 & 0 & -0.15 & -0.15 \\ 0 & 0 & -1 & 0.75 & 0 & 1.1 \end{bmatrix};$$

$$\boldsymbol{\delta} = \begin{bmatrix} 0.19 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.06 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.14 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.43 \end{bmatrix};$$

$$\mathbf{M} = \begin{bmatrix} 0 & 0 \\ -0.575 & 0 \\ 0 & 0 \\ 0 & -0.626 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}; \qquad \mathbf{P} = \begin{matrix} V_1 \\ f_1 \\ V_2 \\ f_2 \end{matrix} \begin{bmatrix} 0.2969 & 0 \\ -0.6277 & 0 \\ 0 & -0.12 \\ 0 & 0.0527 \end{bmatrix}.$$

Using the predictive matrix **P**, we obtain power functions, based on which the optimal values of cutting modes are determined

$$V'_{10} = V_{10} q_1^{0.2969}; \quad f'_{10} = f_{10} q_1^{-0.6277}; \\ V'_{20} = V_{20} q_2^{-0.12}; \quad f'_{20} = f_{20} q_2^{0.0527}. \tag{2.57}$$

To determine the values of cost price C_2^e and optimal cutting modes for a new set of parameters $p_i = \{d_1 = 4{,}5;\ d_2 = 0{,}5\}$, it is enough, without solving the optimization problem, to use the dependencies (2.56) and (2.57)

$$C'_0 = C_0 \left(\frac{d'_1}{d_1}\right)^{0.1948} \left(\frac{d'_2}{d_2}\right)^{0.124} = 10.17 \cdot \left(\frac{4.5}{4.0}\right)^{0.1948} \left(\frac{0.5}{1.0}\right)^{0.124} = 9.46 \text{ cent};$$

$$V'_{10} = 56.66 \cdot \left(\frac{4.5}{4.0}\right)^{0{,}2969} = 58.68 \text{ m/min}; \; f'_{10} = 1.63 \text{ mm/rev};$$

$$V'_{20} = 275.18 \text{ m/min}; \quad f'_{20} = 0.098 \text{ mm/rev}.$$

The optimal values of the objective function and variable cutting modes for the same case of the depth distribution for two-pass machining are presented in Table 2.1. A rather close approximation of the data obtained by the express method (discrepancy of 1.5% for the objective

function and up to 4% for $\{f_1,\ f_2,\ V_2\}$) indicates the effectiveness of this approach.

Table 2.1

Comparison of optimization results by GP method and express method

d_1,	d_2,	C_2,	C_2^e,	V_{10},	f_{10},	V_{20},	f_{20},	V_{10}^e,	f_{10}^e,	V_{20}^e,	f_{20}^e,
mm		cent		m/s	$\frac{mm}{rev}$	m/s	$\frac{mm}{rev}$	m/s	$\frac{mm}{rev}$	m/s	$\frac{mm}{rev}$
2.5	2.5	10.35	10.29	0.55	2.92	4.12	0.1	0.82	2.36	3.78	0.11
2.7	2.3	10.4	10.34	0.6	2.68	4.1	0.1	0.84	2.25	3.82	0.11
3.0	2.0	10.4	10.38	0.68	2.38	4.1	0.1	0.87	2.11	3.88	0.11
3.5	1.5	10.4	10.32	0.8	2.01	4.1	0.1	0.91	1.91	4.1	0.1
4.0	1.0	10.1	10.07	0.91	1.76	4.2	0.1	0.91	1.76	4.22	0.1
4.2	0.8	9.95	9.88	1.0	1.68	4.27	0.1	0.98	1.63	4.59	0.1
4.5	0.8	9.61	9.46	0.11	1.59	4.44	0.1	0.98	1.63	4.59	0.1
4.7	0.3	9.0	8.95	1.2	1.58	4.66	0.1	0.99	1.59	4.88	0.1
4.9	0.1	7.93	7.87	1.35	1.57	5.21	0.1	1.0	1.55	5.56	0.09
4.95	0.05	7.29	7.24	1.43	1.61	5.61	0.11	1.0	1.54	6.05	0.09
d = 5mm			C = 7.44 cent			V = 5.205 m/s			f = 0.11 mm/rev		

To study the behavior of the dependence $C_2 = f(d_2)$, the values of the objective function and cutting elements are calculated in the range of changes in the depths of the finishing pass from 2.5 to 0.05 mm [41-43] (Table 2.1) using the GP method C_2 and the express method C_2^e. Analysis of tabular data indicates an acceptable approximation of express estimates (except for the values of the variable V_1). The function $C_2 = f(d_2)$ is convex, monotonically decreasing in the direction of the d_2 axis and does not have a global extremum in the entire domain of definition.

The lower bound of the cutting depth d_2 will coincide with the calculated minimum finishing allowance, which for this case is $d_2 = 0.138$ mm. The real value of the allowance, determined by the industry tables, is $d_2 = 0.8$ mm. The optimal modes of two-pass machining in this depth range d_2 are presented in the table. At the lower boundary of this range, the cost price values of two-pass and single-pass machining are commensurate. Besides, in a wider range, there is one extreme point of maximum cost ($d_2 = 1.3$ mm), which should be avoided when assigning cutting depths.

The proposed express procedure for optimizing cutting modes using the GP method allows us to solve the optimization problem of two-pass machining, while the computational complexity is significantly reduced, and it becomes possible to analyze the objective cost price function in a wide range of changes in cutting depths.

3. METHOD OF LAGRANGE MULTIPLIERS

3.1. General Considerations

When searching for optimal technological solutions in the problems of technological preparation of production, one has to use both numerical and analytical methods for solving extreme problems. The first ones make it possible to quite efficiently solve complex optimization problems of large dimensions, which are almost impossible to solve by analytical methods. However, the latter has the advantage of being able to determine a qualitative picture of the optimal solution behavior when changing the structure and parameters of the problem. Currently, intensive attempts are being made to combine these methods in the development of human-machine procedures for extrema searching, in which analytical tools are used to analyze the results of the numerical solution of the subtasks for the initial optimization problem.

It should be noted that when searching for a conditional extremum of a function $z = f(x_1, x_2, ..., x_n)$ of variables, the main method for the analytical solution of the problem is the method of Lagrange multipliers and its generalization [44, 45].

Suppose that you want to minimize the function $z = f(\bar{x})$ where the variable $\bar{x}$ is constrained $g_1(\bar{x}) = 0,\ \ g_2(\bar{x}) = 0,\ \ g_m(\bar{x}) = 0.$ The indicated functions have partial derivatives of the first and second order in all $x_i \in \bar{x}$. Then, to solve the problem, it is necessary to compose the

Lagrange function $F\left(\bar{x},\bar{\lambda}\right)=f\left(\bar{x}\right)+\sum_{i=1}^{m}\lambda_i g_i\left(\bar{x}\right)$ and solve the system of equations:

$$\begin{cases} \dfrac{\partial F}{\partial x_j}=\dfrac{\partial f}{\partial x_j}+\sum\limits_{i=1}^{m}\lambda_i\dfrac{\partial g_i}{\partial x_i}=0 & when\ \ j=1,2,...,n; & (3.1) \\ \dfrac{\partial F}{\partial \lambda_i}=g_i\left(\bar{x}\right)=0 & when\ \ \ i=1,2,...,m. & (3.2) \end{cases}$$

These equations are a necessary condition for the extremum of the function $z=f\left(\bar{x}\right)$. Sufficient conditions for the minimum are equations (2.49) and (2.50), as well as the positive definiteness of the quadratic form:

$$\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial x_i \partial x_j}dx_i dx_j,$$

i.e. second differential of the Lagrange function.

The necessary conditions for the minimum function $z=f\left(\bar{x}\right)$ in the presence of constraints in the form of inequalities $g_i\left(\bar{x}\right)\le b_i\left(i=\overline{1,m}\right)$ are as follows:

$$\begin{cases} \dfrac{\partial F}{\partial x_j}+\sum\limits_{i-1}^{m}\lambda_i\dfrac{\partial g_i}{\partial x_j}=0 & when\ \ j=1,2,...,n; \\ g_i(x)\le b_i & when\ \ i=1,2,...,m; \\ \lambda_i[g_i(x)-b_i]=0 & when\ i=1,2,...,m; \\ \lambda_i\ge 0 & when\ \ i=1,2,...,m. \end{cases}$$

These conditions are known as Kuhn-Tucker conditions. If $f\left(\bar{x}\right)$ and $g_i\left(\bar{x}\right)$ $\left(i=\overline{1,m}\right)$ are convex functions, then the Kuhn-Tucker conditions are also sufficient.

3.2. Single-pass optimization

Let the objective function:

$$C=\frac{a+bV^m S^n t^k}{VS};\quad b=\frac{a\ t_{ch}+a'}{k_t C_t},$$

where C – variable part of the machining cost price; a - the cost price of a machine-minute; a' – reduced cost of the instrument.

An analysis of the constraints showed that it is necessary to take into account the permissible tangential component of the cutting force $P_{z\ pm}$ and the kinematically achievable cutting speed and feed. Then, the task of optimizing cutting modes is described by the following mathematical model:

$$C=\frac{a+bV^m f^n d^k}{Vf}\rightarrow \min;$$

$$P_z=k_{pz}C_{pz}V^{\alpha_1}f^{\alpha_2}d^{\alpha_3}\le P_{z\ pm}$$

$$V_{\min}\le V\le V_{\max};$$

$$f_{\min}\le f\le f_{\max}.$$

To find the optimum, we use the Kuhn-Tucker conditions. To do this, we rewrite the kinematic constraints in the form $-V \le V_{\min}$, $V \le V_{\max}$, $-f \le f_{\min}$, $f \le f_{\max}$ and compose the Lagrange Function:

Consider the necessary minimum conditions:

$$\begin{cases} \dfrac{\partial \Phi}{\partial V} = \dfrac{bV^m f^n d^k (m-1) - a}{V^2 f} + \lambda_1 k_{pz} C_{pz} \alpha_1 V^{\alpha_1 - 1} f^{\alpha_2} d^{\alpha_3} - \lambda_2 - \lambda_3 = 0; & (3.3) \\ \dfrac{\partial \Phi}{\partial S} = \dfrac{bV^m f^n d^k (n-1) - a}{V f^2} + \lambda_1 k_{pz} C_{pz} \alpha_2 V^{\alpha_1} f^{\alpha_2 - 1} d^{\alpha_3} - \lambda_4 - \lambda_5 = 0 & (3.4) \\ \lambda_1 \dfrac{\partial \Phi}{\partial \lambda_1} = \lambda_1 (k_{pz} C_{pz} V^{\alpha_1} f^{\alpha_2} d^{\alpha_3} - P_{z\,pm}) = 0; & \\ \lambda_2(-V + V_{\min}) = 0; \quad \lambda_3(V - V_{\max}) = 0; \quad \lambda_4(-f + f_{\min}) = 0; & \\ \lambda_5 (f - f_{\max}) = 0; \; k_{pz} C_{pz} V^{\alpha_1} f^{\alpha_2} d^{\alpha_3} \le P_{z\,pm}; & (3.5) \\ -V \le -V_{\min}; \; V \le V_{\max}; \; -f \le -f_{\min}; \; f \le f_{\max}. & \end{cases}$$

We try to find solutions to these equations.

1. If $V_{\min} < V < V_{\max}$, $f_{\min} < f < f_{\max}$, then $\lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0$.

When $\lambda_1 = 0$ the system of equations (3.3) and (3.4) does not have a solution that satisfies the physics of the process. Let $\lambda_1 \ne 0$ then:

$$k_{pz} C_{pz} V^{\alpha_1} f^{\alpha_2} d^{\alpha_3} = P_{z\,pm}.$$

From (3.3) and (3.4) we obtain:

$$\frac{bV^m f^n d^k (m-1) - a}{V^2 f} + \frac{\lambda_1 \alpha_1 P_{z\,pm}}{V} = 0;$$
$$\frac{bV^m f^n d^k (n-1) - a}{V f^2} + \frac{\lambda_1 \alpha_2 P_{z\,pm}}{f} = 0.$$

We solve the resulting system of equations:

$$\alpha_2 bV^m f^n d^k (m-1) - \alpha_2 a - \alpha_1 bV^m f^n d^k (n-1) + \alpha_1 a = 0;$$
$$V^m = \frac{a(\alpha_1 - \alpha_2) d^{-k} f^{-n}}{b(\alpha_2 (m-1) - \alpha_1 (n-1))} = \ell_1^m d^{-k} f^{-n}, \qquad (3.6)$$

where $\ell_1^m = \dfrac{a(\alpha_1 - \alpha_2)}{b(\alpha_2 (m-1) - \alpha_1 (n-1))}$.

Substituting (3.6) into (3.5), we obtain:

$$k_{pz} C_{pz} \ell_1^m d^{-\frac{k\alpha_1}{m}} f^{-\frac{n\alpha_1}{m}} f^{\alpha_2} d^{\alpha_3} = P_{z\,pm};$$
$$f^{\frac{\alpha_2 m - \alpha_1 n}{m}} = \frac{P_{z\,pm}\, d^{\frac{\alpha_1 k - \alpha_3 m}{m}}}{k_{pz} C_{pz} \ell_1^{\alpha_1}};$$
$$f = P_{z0}^{1/x} d^{y/x} \ell_1^{-\alpha_1/x}, \qquad (3.7)$$

where $P_{z0} = \dfrac{P_{z\,\mathrm{pm}}}{k_{pz} C_{pz}}$; $x = \dfrac{\alpha_2 m - \alpha_1 n}{m}$; $y = \dfrac{\alpha_1 k - \alpha_3 m}{m}$.

Substituting (3.6) into (3.7), we obtain:

$$V = \ell_1 d^{-\frac{k}{m}} P_{z0}^{-\frac{n}{m}} d^{\frac{ny}{xm}} \ell^{\frac{\alpha_1 n}{xm}} = P_{z0}^{-\frac{n}{mx}} d^{\frac{kx + ny}{mx}} \ell_1^{\frac{xm + \alpha_1 n}{mx}}.$$

Thus, the optimal values of the cutting speed and feed:

$$V_{01} = \left(P_{z0}^{-n} e^{mx - \alpha_{1n}} d^{-(kx - ny)} \right)^{\frac{1}{mx}};$$

$$f_{01} = \left(P_{z0} \ell_1^{-\alpha_1} d^{y} \right)^{\frac{1}{x}}.$$

2. Let λ_2, λ_3, $\lambda_4 = 0$ and $\lambda_5 \neq 0$. Then $f_0 = f_{\max}$.

For $\lambda_1 = 0$ from (3.3) we have:

$$bV^m f_{\max}^n d^k (m-1) - a = 0;$$

$$V_{02} = \left(\frac{a}{b(m-1)} f_{\max}^{-n} d^{-k} \right)^{\frac{1}{m}}.$$

For $\lambda_1 \neq 0$ from (3.5) we have:

$$k_{pz} C_{pz} V^{\alpha_1} f_{\max}^{\alpha_2} d^{\alpha_3} = P_{z\,pm};$$

$$V_{03} = \left(P_{z0} f_{\max}^{-\alpha_2} d^{-\alpha_3} \right)^{\frac{1}{\alpha_1}}.$$

3. Let $\lambda_2, \lambda_4, \lambda_5 = 0;$ $\lambda_3 \neq 0.$ Then $V_0 = V_{\max}$.

At $\lambda_1 = 0,$ $f_{02} = \left(\frac{a}{b(n-1)} V_{\max}^{-m} d^{-k} \right)^{\frac{1}{n}}.$

At $\lambda_1 \neq 0,$ $f_{03} = \left(P_{z0} V_{\max}^{-\alpha_1} d^{-\alpha_3} \right)^{\frac{1}{\alpha_2}}.$

It can be shown that the remaining cases are inappropriate to consider.

Thus, we have how many points satisfying the necessary conditions-equalities. The global minimum is found by comparing the values of the objective functions at the points with the coordinates (V_{01}, f_{01}); $(V_{02}, f_{\max})$; $(V_{03}, f_{\max})$; $(V_{\max}, f_{02})$; $(V_{\max}; f_{03})$ and only those points that satisfy the

inequality conditions are compared. The analysis showed that in practice one can be satisfied by comparing only points 1 and 2.

Using the Lagrange multiplier method in the task of optimizing cutting conditions during roughing [31]

Let it be necessary to machine a part of the “axis” type l = 800 mm long, diameter D = 80 mm, made of constructional steel. External longitudinal turning was performed on a screw-cutting lathe 1K62 (n_{min} = 12 rpm, n_{max} = 2000 rpm, f_{min} = 0.07 mm/rev, f_{max} = 4.16 mm/rev, P_e = 10 kW). Choose the material of the cutting part DIN HS 123 (T15K6 hard alloy) and the geometric parameters of the tool cutting part (φ = 45^0, γ = 10^0). For the convenience of calculation, we assume what are the correction factors $k_t = k_{pz} = 1$.

Let the calculation of the tool holder strength for the cutter and tip, the maximum allowable torque on the spindle of the machine $P_{z\,pm} = 3000\ N$ is selected. From the general engineering tables we set the values all other parameters: $a = 2$ cent / min; $a' = 15$ cent/life time; $m = 5$; $n = 2.25$; $k = 1$; $C_{pz} = 300$; $\alpha_1 = -0.15$; $\alpha_2 = 0.75$; $\alpha_3 = 1$; $d = 3$ mm.

As a result, we get:

$V_{01} = 113.2$ m/min; $f_{01} = 0.6$ mm/rev; $V_{02} = 42.7$ m/min; $f_{02} = f_{\max} = 4.16$ mm/rev.

In rough, in addition to the force constraint, the constraint on the allowable cutting power $N_c \geq \frac{P_z V}{60 \cdot 1020}$ may be active: or at the same time a pair of constraints;

$$\begin{cases} P_z \ge k_{pz} C_{pz} V^{\alpha_1} f^{\alpha_2} d^{\alpha_3} \\ N_c \ge \dfrac{P_z V}{60 \cdot 1020} \end{cases} \quad \text{or} \quad \begin{cases} f \le f_{\max} \\ N_c \ge \dfrac{P_z V}{60 \cdot 1020} \end{cases}.$$

In the latter case, the optimal values of the cutting modes are found as solutions to these pairs of equations.

By analyzing the equation $C = \dfrac{a + bV^m f^n d^k}{Vf}$, it can be shown that the set of optimal solutions lies on the boundary of admissible values, and only inequality is taken into account from parametric constraints.

Thus, to determine the optimal cutting modes for rough single-pass machining, it is practical enough to consider five alternative options and choose the best one from them (fig. 3.1).

Instead of power drive constraint, we should analyze the constraint to a permissive surface roughness during finish machining. Also, it must be remembered that during finishing, the tool life should be a multiple of the cutting time when machining one part, i.e. tool replacement is allowed only after complete machining of the part. In this case, it is necessary to correct the optimal cutting modes. For example, let the cutting force constraint be active. The optimal values of the cutting modes determine the tool life of 24 min., and the cutting time is 5 min.

Then, if it is necessary to strictly comply with the constraint on the cutting force, you must either reduce the tool life, leaving the cutting time unchanged or increase the cutting time, leaving the tool life unchanged. Then compare the results with the objective function. However, these solutions will not lie on the region border of admissible values. To eliminate this drawback, it is necessary to use the equation $T = k\,t_m$, where k – an integer equal to the number of parts processed during the tool life.

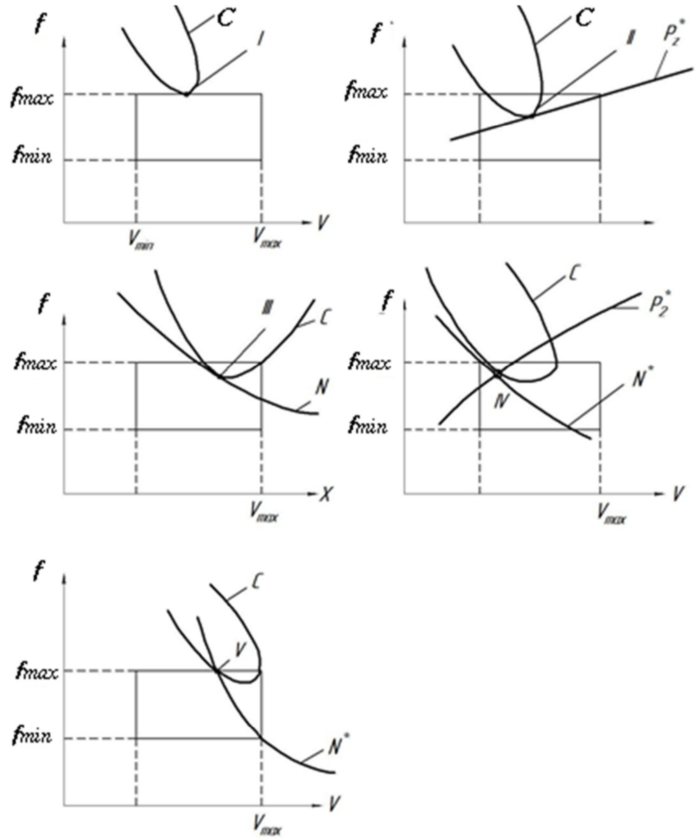

Fig. 3.1. Alternative options for calculating the optimal rough-cutting modes

Solving the equations together:

$$P_{z\,pm} = k_{pz} C_{pz} V^{\alpha_1} f^{\alpha_2} d^{\alpha_3} \quad \text{and} \quad T = k\,t_m,$$

two pairs $<V_1, f_1>$ and $<V_2, f_2>$ can be defined between which a point $<V_0, f_0>$ lies on the line of $P_{z\,pm}$ restriction. Comparing the obtained solutions by the value of the objective function, we choose the best one from them.

Using the Lagrange multiplier method in the task of optimizing cutting modes during finishing

It is required to determine the optimal cutting modes for finishing turning of a smooth shaft made of C45 (steel 45): d = 100 mm, l = 800 mm) on a 16K20 machine with a tool – lathe tool cutter with the mechanical fastening of a hexagonal tool tip made of DIN HS 123 (T15K6 hard alloy). The cutting parameters are determined by the values: a' = 10 cent, r = 1 mm, $\varphi = 45^0$, $\varphi_1 = 10^0$, $\gamma = 10^0$, b × h = 25 × 25 mm, l_t = 50 mm, h_n = 5 mm. Turning is carried out in centers with a cutting depth of d = 0.5 mm, a roughness of Ra. = 2.5 µm, and precision of diametrical size of 10 quality class (the size error is 0.12 mm).

Kinematics of the machine allows the feed of the bed carriage (mm/rev) in the range of $0.05 \le f \le 2.8$ and the spindle speed in the range of $12.5 \le n \le 1600$. The cutting force allowed by the strength of the feed mechanism is P_{fpm} = 6000 N; the main drive power is 10 kW; the drive efficiency is η = 0.8; the flexibility of the lathe system in the weakest section is $w_{\max}$ = 0.096 µm/N [5], the cost of the machine-minute a = 1 cent, tool change time t_{ch} = 2 min.

The cost of the instrument will be calculated by the formula:

$$T = \frac{C_T k_T}{V^m f^n d^k},$$

where $m = 5$; $n = 1$; $k = 0.45$; $C_T = 420^5 = 1.31 \cdot 10^{13}$; $k_T = 1$.

Cutting force is determined from the expression

$$P_z = C_{pz} k_{pz} V^{\alpha_1} f^{\alpha_2} d^{\alpha_3},$$

where $C_{pz} = 300$; $k_{pz} = 1$; $\alpha_1 = -0.15$; $\alpha_2 = 0.75$; $\alpha_3 = 1$.

We form the region of permissive values of the cutting modes. To do this, we define a set of constraints imposed on cutting modes.

1. Constraint on the power of the electric motor (N_e, kW) drive the main movement of the machine

$$N_c = \frac{P_z \cdot V}{60 \cdot 1020} = \frac{V^{0.85} f^{0.75} d^1}{20.4} \le N_e \cdot \eta = 10 \cdot 0.8 = 8;$$
$$V f^{0.75} d^1 \le 163.2.$$

2. The constraint on the force allowed by the strength of the weak link of the machine feed mechanism:

$$P_x \le P_{pm}; \; P_x = 0.25 P_{pm} \to P_z \le 4 P_{pm} = 24000H.$$

3. The constraint on the strength of the tool holder

$$M_b = P_z l_c \le \frac{\sigma_b W_c}{K_m}; \quad W_c = \frac{b \cdot h^2}{6},$$

where M_b – bending moment from the force; l_c – the cutter overhang; σ_b – tensile strength of the cutter in bending; W_c – a moment of the cutter holder section resistance; K_m – safety margin (K_m = 1.5 ... 2).

We take σ_b = 600 MPa, K_m = 1.5. Then:

$$300 \cdot V^{-0.15} \cdot f^{0.75} \cdot d^1 \cdot 50 \le \frac{600 \cdot 25 \cdot 25^2}{1.5 \cdot 6} \to P_z \le 20830N.$$

4. The constraint on the strength of the cutting tip:

$$P_z \le 34 \cdot d^{0.77} h_{ct}^{1.25} \left(\frac{\sin 60^0}{\sin \varphi} \right)^{0.8} = 34 \cdot 0.5^{0.77} \cdot 5^{1.35} \cdot 1.5^{0.8} = 6000\ N,$$

where h_{ct} – thickness of the carbide cutting tip.

5. The constraint on the permissible rigidity of the cutter:

$$P_z \le \frac{3 \cdot f_c E\ J}{l_c^3} = \frac{3 \cdot 0.05 \cdot 2.1 \cdot 10^5 \cdot 25 \cdot 25^3}{50^3 \cdot 12} = 13020\ N,$$

where f_c – permissible deflection of the cutter, mm; $J = \frac{bh^2}{12}$ – the polar moment of inertia of the section of the tool holder, mm^3; E – modulus of elasticity of the tool holder, MPa.

6. The constraint on the rigidity of the detail. This constraint takes into account the maximum allowable deflection of the detail, which is set depending on the tolerance Δ_j on the size of the detail: $f_d = (0.25 \ldots 0.5) \cdot \Delta j$

Deflection of a part under the action of a concentrated force R: $R = \sqrt{P_z^2 + P_y^2} \approx 1{,}1 P_z$ is equal to:

$$f_d = \frac{R\ L_d^3\ \mu}{K_{fd} E\ J_d},$$

where L_d – distance from the support to the section under consideration, for this calculation option, we take L_d = 400 mm; J_d – a polar moment of inertia, mm^4; μ – dynamic coefficient; K_{fd} – coefficient depending on the method of fastening the detail (K_{fd} = 2.4 ... 3 – when fastening the detail

cantilever in the chuck, K_{fd} = 70 ... 100 when fastening in the centers, K_{fd} = 130 ... 140 when fixing in the chuck with preloading the center rear tailstock). In our case, the constraint has the form:

$$P_z \le \frac{f_d\ K_{fd} E\ J_d}{1{,}1 \cdot L_d^3 \cdot \mu} = \frac{0.03 \cdot 100 \cdot 2.1 \cdot 10^5 \cdot 0.05 \cdot 100^4}{1.1 \cdot 400^3 \cdot 1.2} = 3550\ N.$$

7. The constraint on the roughness of the machined surface. We use the empirical equation [7]:

$$R_a = 0.16 f^{0.59} V^{-0.19} r^{-0.29} (90 + \gamma)^{0.86} \le R_{a\ pm}$$

and theoretical dependence:

$$R_z = r - \sqrt{r^2 - \left(\frac{f}{2} \right)^2}.$$

The actual values of R_z are approximately 1.4 times more than theoretically calculated. Taking $R_z = 5R_a$, we get:

$$f_{\max} = 2\sqrt{(1 - R_z)^2 - 1} \approx 0.36 \text{ mm/rev.}$$

Thus, a mathematical model for calculating the optimal cutting modes will be:

$$C = \frac{1+\frac{12+10}{1.07\cdot 10^{13}} V^5 f^{0.75} d^1}{V\cdot f} \to \min;$$

$$P_z = 300\ V^{-0.15} f^{0.75} d^1 \le 3550 \to V^{-0.15} s^{0.75} t^1 \le 11.8;$$

$$\frac{P_z \cdot V}{1020\cdot 60} \le 8\ \text{kW} \to \mathrm{V}^{0.85} f^{0.75} d^1 \le 1632;$$

$$R_a = 3.55\ V^{-0.19} f^{0.59} \le 2.5 \to v^{-0.19} s^{0.59} \le 0.75;$$

$$f \le 2.8;\ f \ge 0.05;\ V \le 502.4;\ V \ge 3.9.$$

We calculate the optimal cutting mode with active force constraint:

$$x = \frac{0.75\cdot 5 - (-0.15)\cdot 1}{5} = 0.78;\quad y = \frac{(-0.15)\cdot 0.75 - 1\cdot 5}{5} = -1.02;$$

$$\ell_1^m = \frac{a(\alpha_1 - \alpha_2)}{b(\alpha_2(m-1) - \alpha_1(n-1))};$$

$$\ell_1 = \left(\frac{1\cdot(0.75-(-0.15))}{9.18\cdot 10^{13}\,(0.75\cdot(5-1)) - (-0.15\cdot(1-1))}\right)^{\frac{1}{5}} = 200.84;$$

$$f_0 = \left(1{,}18\cdot 200.84^{0.15}\cdot 0.5^{-1.02}\right)^{\frac{1}{0.78}} = 8.34\ \text{mm/rev};$$

$$\mathrm{V}_0 = \left(1.18^{-1}\cdot 200.84^{3{,}75}\cdot 0.5^{0.435}\right)^{\frac{1}{0.78\cdot 5}} = 145.18\ \text{m/min}.$$

When determining the value ℓ_1, dependence (3.6) was used.

With an active power constraint, we have:

$$x = \frac{0.75\cdot 5 - 0.85\cdot 1}{5} = 0.58;\quad y = \frac{0.85\cdot 0.75 - 1\cdot 5}{5} = -0.87;$$

$$\ell_1 = \left(\frac{1\cdot(0.75-0.85)}{9.18\cdot 10^{13}\,(0.75\cdot(5-1)) - (0.85\cdot(1-1))}\right)^{\frac{1}{5}} = -3.63^{0.2}\cdot 10^2 < 0.$$

The last inequality means that the points of tangency of the level lines of the objective function and line of constraint on the permissible drive power of the main motion do not exist.

Consider the surface roughness constraint:

$$x = \frac{0.59\cdot 5 - (-0.19)}{5} = 0.628;\quad y = \frac{(-1.19)\cdot 0.75 - 0.5}{5} = -0.03;$$

$$\ell_1 = \left(\frac{1\cdot(0.59-(-0.19))}{9.18\cdot 10^{13}\,(0.59\cdot(5-1)) - (-0.19\cdot(1-1))}\right)^{\frac{1}{5}} = 204.77;$$

$$f_0 = \left(0.75\cdot 204.77^{0.19}\cdot 0{,}5^{-0.03}\right)^{\frac{1}{0.628}} = 3.16\ \text{mm/rev};$$

$$\mathrm{V}_0 = \left(0.75^{-1}\cdot 204.77^{2.95}\cdot 0.5^{-0.47}\right)^{0.32} = 185.97\ \text{m/min}.$$

We calculate the optimal cutting speed for $f = f_{max} = 0.36$ mm/rev. Here f_{max} is the maximum allowable feed when turning with a roughness $R_a = 2.5$ [6]:

$$V_0 = \left(\frac{1}{9.18\cdot 10^{13}\cdot 4}\cdot 0.36^{-1}\cdot 0.5^{-0{,}75}\right) = 263.58\ \text{m/min}.$$

We calculate the coordinates of the intersection point for the constraint lines in f_{max} and N_{pm}. To do this, solve the system of equations:

$$\begin{cases} f = f_{max} = 0.36; \\ V^{0.85} f^{0.75} d^{1.0} = 163,2 \end{cases} \rightarrow V = \left(\frac{326.4}{0.36^{0.75}} \right)^{\frac{1}{0.85}} = 223.2 \text{ m/min.}$$

We define the coordinates of the intersection point of the restriction lines in $P_{z\,pm}$ and N_{pm}:

$$\begin{cases} V^{-0.15} f^{0.75} d^{1} = 0.69; \\ V^{0.85} f^{0.75} d^{1} = 163.2 \end{cases} \rightarrow f = \begin{cases} \left(1,38 V^{0.15}\right)^{\frac{1}{0.75}} = 4.57 \text{ mm/rev;} \\ V = \dfrac{326.4}{1.38} = 236.5 \text{ m/min.} \end{cases}$$

Let the calculations show that the best solution that satisfies all the constraints is a pair: {f = 0.36 mm/rev; V= 264 m/min). We will correct the cutting modes. The closest lower feed value allowed by the machine kinematics is f = 0.35 mm/rev. Then the optimal value of the cutting speed: V = 265 m/min (n = 844 min^{-1}). The closest rotation speed: n_1 = 800 min^{-1}, n_2 = 1000 min^{-1}. We calculate the cost price of the technological pass at these two points:

$$C_1 = \frac{1 + 9.18 \cdot 10^{-13} \cdot 0.35^1 \cdot 0.5^{0,75} \cdot 251^5}{0.35 \cdot 251} = 21.67;$$

$$C_2 = \frac{1 + 9.18 \cdot 10^{-13} \cdot 0.35^1 \cdot 0.5^{0,75} \cdot 314^5}{0.35 \cdot 314} = 53.05.$$

The calculation result shows that the optimal values of the cutting modes: {f_0 = 0.35 mm/rev, n_0 = 800 min^{-1}}.

If the cutting speed changed continuously, as, on modern CNC machines, the correction of the modes should be carried out according to the number of parts processed for one period of tool life. We perform the necessary transformations:

$$T = k\,t_m \rightarrow \frac{C_T k_T}{V^m f^n d^k} = \frac{k\pi D l_d}{1000Vf} \rightarrow V^{m-1} = \frac{C_T k_T \cdot 1000}{k\pi D l_d f^{n-1} d^k} = \frac{1.307 \cdot 10^{12}}{k \cdot 3.14 \cdot 8 \cdot 0.59} =$$

$$= \frac{8.8 \cdot 10^{10}}{k} \rightarrow k = \frac{8.8 \cdot 10^{10}}{264^4} \sqrt{b^2 - 4ac}.$$

We select such values k that approximate V the smallest value above and below. At k = 18, the cutting speed V= 254.4 m/min; at k = 17, the cutting speed V = 268.3 m/min. Lower cost price corresponds to the first value V = 254.4 m/min. Then the corrected cutting mode corresponds to {f = 0.35 mm/rev; V = 254.4 m/min}.

3.3. Two-pass optimization by Lagrange multiplier method

In rough operations, machining allowance is often removed in several passes. When two-pass processing, the cost price of the technological step for turning the cylindrical surface is determined from the equation

$$C_{\mathrm{II}} = a\left[t_m^{\mathrm{I}} + t_m^{\mathrm{II}} + t_a + t_a^{'} + t_{ch}\left(\frac{t_m^{\mathrm{I}}}{T^{\mathrm{I}}} + \frac{t_m^{\mathrm{II}}}{T^{\mathrm{II}}}\right)\right] + a'\left[\frac{t_m^{\mathrm{I}}}{T^{\mathrm{I}}} + \frac{t_m^{\mathrm{II}}}{T^{\mathrm{II}}}\right],$$

where t_m^{I} (t_m^{II}) – cutting time in the first (second) pass; T^{I} $\left(T^{II}\right)$ – tool life at the first (second) pass; t_a – auxiliary time for the step; t_a' – the time of displacement and mounting of the tool to perform the second pass; t_{ch} – tool change time.

Tool life is given by equations

$$T^{\mathrm{I}} = \frac{k_T C_T}{V_1^m f_1^n d_1^k}; \qquad T^{\mathrm{II}} = \frac{k_T C_T}{V_2^m f_2^n d_2^k},$$

where $V_1, f_1, d_1 \, (V_2, f_2, d_2)$ – cutting mode, respectively, in the first and second passes.

Minimizing the cost of two-pass machining reduces to the optimal distribution of the total allowance between passes and to an independent search for the optimal values of the cutting and feed rates at each pass at a fixed cutting depth *d*.

Consider the situation when force constraints were active on the first and second passes [46-48]. Then you can parameters $V_1, f_1;\ V_2, f_2$ substitute their expressions through d_1 and d_2. As a result of transformations, we obtain

$$\frac{a+bV^m f^n d^k}{Vf} = \frac{a+b\left(P_{z0}^{-n}\ell_1^{mx-\alpha_1-n}\, d^{-(kx+ny)}\right)^{\frac{n}{mx}}\left(P_{z0}\ell_1^{-\alpha_1} d^y\right)^{\frac{n}{x}} d^k}{\left(P_{z0}^{-n}\ell_1^{mx+\alpha_1 n}\, d^{-(kx+ny)}\right)^{\frac{1}{mx}}\left(P_{z0}\ell_1^{-\alpha_1} d^y\right)^{\frac{1}{x}}} =$$

$$= \frac{a+bP_{z0}^{-n/x}\,\ell_1^{m+\frac{\alpha_1 n}{x}}\, d^{-\left(k+\frac{ny}{x}\right)} P_{z0}^{\frac{n}{x}}\,\ell_1^{-\frac{\alpha_1 n}{x}}\, d^{\frac{ny}{x}}\, d^k}{P_{z0}^{-n/mx}\,\ell_1^{\frac{\alpha_1 n}{mx}}\, d^{-\frac{kx+ny}{mx}}\, P_{z0}^{\frac{1}{x}}\,\ell_1^{-\frac{\alpha_1}{x}}\, d^{\frac{y}{x}}} =$$

$$= \frac{a+b\ell_1^m}{P_{z0}^{\frac{m-n}{mx}}\,\ell_1^{\frac{\alpha_1(n-m)}{mx}}\, d^{\frac{my-ny-kx}{mx}}} = \frac{a+b\ell_1^m}{\left(P_{z0}^{m-n}\,\ell_1^{\alpha_1(n-m)}\right)^{\frac{1}{mx}} d^z};$$

$$z = \frac{y^{(m-n)-kx}}{mx}.$$

The objective function will take the form:

$$C_{\text{II}} = \frac{a+b\ell_1^m}{\left(P_{z01}^{m-n}\,\ell_1^{\alpha_1(n-m)}\right)^{\frac{1}{mx}} d_1^z} + \frac{a+b\ell_1^m}{\left(P_{z02}^{m-n}\,\ell_1^{\alpha_1(n-m)}\right)^{\frac{1}{mx}} d_2^z}.$$

Given the constraint on the total allowance $d_1 + d_2 = 4$, we can formulate the following mathematical model for optimizing two-pass processing:

$$\begin{cases} d_{\text{II}}^* = \dfrac{1}{P_{z01}^{\frac{m-n}{mx}}\, d_1^z} + \dfrac{1}{P_{z02}^{\frac{m-n}{mx}}\, d_2^z} \to \min; \\ u - d_1 - d_2 = 0. \end{cases}.$$

The Lagrange function will have the form:

$$F = C^* + \lambda(u - d_1 - d_2).$$

Then the optimal cutting depths are found as solutions to the system of equations:

$$\begin{cases} \dfrac{\partial F}{\partial d_1} = \dfrac{-z}{P_{z01}^{\frac{m-n}{mx}}\, d_1^{z+1}} - \lambda = 0 \\ \dfrac{\partial F}{\partial d_2} = \dfrac{-z}{P_{z02}^{\frac{m-n}{mx}}\, d_2^{z+1}} - \lambda = 0 \\ \dfrac{\partial F}{\partial \lambda} = u - d_1 - d_2 = 0 \end{cases} \quad \Bigg| \to \quad \left(\frac{d_1}{d_2}\right)^{z+1} = \left(\frac{P_{z02}}{P_{z01}}\right)^{\frac{m-n}{mx}}; \ \downarrow \ d_1 = \upsilon_1 d_2; \quad \upsilon = \left(\frac{P_{z02}}{P_{z01}}\right)^{\frac{m-n}{mx(z+1)}};$$

$$\mathrm{u} - \upsilon_1 \mathrm{d}_2 - \mathrm{d}_2 = 0 \to \mathrm{d}_2 = \frac{\mathrm{u}}{1+\upsilon_1}; \quad d_1 = \frac{\upsilon_1 u}{1+\upsilon_1}.$$

Let us analyze the dependence of the two-pass machining cost price on the depth of cut at the first step. To do this, take the second derivative of the function C_{II}^*:

$$C^*_{\text{II}} = \frac{1}{P_{z01}^{\frac{m-n}{m\,x}}\, d_1^z} + \frac{1}{P_{z02}^{\frac{m-n}{m\,x}}\,(u-d_1)^z}.$$

We get:

$$\frac{\partial^z C^*_{\text{II}}}{\partial d_1^z} = \frac{z(z+1)}{P_{z01}^{\frac{m-n}{m\,x}}\, d_1^{z+2}} + \frac{z(z+1)}{P_{z02}^{\frac{m-n}{m\,x}}\,(u-d_1)^{z+2}}.$$

From this equation it is seen that if $z(z+1) > 0$, then the stationary point $d_1 = \frac{\upsilon_1 u}{1+\upsilon_1}$ is a minimum point. If $z(z+1) < 0$, then the stationary point is the maximum point. In the latter case, due to the absence of other stationary points, we will look for a minimum at the boundary of admissible values. Here it is necessary to consider and compare two options $d_1 = d_{pm}$, $d_2 = u - d_{pm}$ and $d_2 = d_{pm}$, $d_1 = u - d_{pm}$. The cost price of the first option is less than the cost price of the second if the inequality holds for:

$$\frac{1}{P_{z01}^{\frac{m-n}{m\,x}}\, d_{pm}^z} + \frac{1}{P_{z02}^{\frac{m-n}{m\,x}}\left(u-d_{pm}\right)^z} - \frac{1}{P_{z01}^{\frac{m-n}{m\,x}}\left(u-d_{pm}\right)^z} + \frac{1}{P_{z02}^{\frac{m-n}{m\,x}}\, d_{pm}^z} < 0.$$

After the transformations we get:

$$\left(P_1^{\frac{m-n}{m\,x}} - P_2^{\frac{m-n}{m\,x}}\right)\left(d_{pm}^z - \left(u-d_{pm}\right)^z\right) < 0.$$

Since $P_1 > P_2,\ d_{\partial on} > (u - d_{pm})z < 0$, then the indicated inequality always holds.

The practical conclusion [49-50] from the analysis is as follows: if force constraints are active $z(z+1) > 0$ during two-pass processing, then the cutting depth in the first pass is calculated by the formula $d_1' = \frac{\upsilon_1 u}{1+\upsilon_1}$; if $z(z+1) < 0$, then the depth of cut in the first pass is taken to be the maximum allowable from technological considerations.

Consider the case when constraints on the type $f_1 \le f_{1\max}$ and $f_2 \le f_{2\max}$ are active on the first and second passes. Then the objective function takes the form:

$$C_{\text{II}} = \frac{a + b\frac{a}{b(m-1)} f_1^{-n} d_1^{-k} f_1^n d_1^k}{\left(\frac{a}{b(m-1)}\right)^{\frac{1}{m}} f_1^{-\frac{n}{m}} d_1^{-\frac{k}{m}} f_1} + \frac{a + b\frac{a}{b(m-1)} f_2^{-n} d_2^{-k} f_2^n d_2^k}{\left(\frac{a}{b(m-1)}\right)^{\frac{1}{m}} f_2^{-\frac{n}{m}} d_2^{-\frac{k}{m}} f_2} =$$

$$= \frac{a + \frac{a}{m-1}}{\left(\frac{a}{b(m-1)}\right)^{\frac{1}{m}}}\left(\frac{1}{f_1^{1-\frac{n}{m}} d_1^{-\frac{k}{m}}} + \frac{1}{f_2^{1-\frac{n}{m}} d_2^{-\frac{k}{m}}}\right) =$$

$$= \left(a^{m-1} b(m-1)^{1-m}\right)^{\frac{1}{m}} m\left(\frac{1}{f_1^{\frac{m-n}{m}} d_1^{-\frac{k}{m}}} + \frac{1}{f_2^{\frac{m-n}{m}} d_2^{-\frac{k}{m}}}\right).$$

Given the constraint $d_1 + d_2 = u$ and using the method of Lagrange multipliers, we obtain the following expressions for determining the optimal cutting depths in the passes:

$$d_1 = \upsilon_2 d_2, \quad \upsilon_2 = \left(\frac{f_{2\max}}{f_{1\max}}\right)^{\frac{n-m}{k}}; \quad d_1 = \frac{\upsilon_2 u}{1+\upsilon_2}; \quad d_2 = \frac{u}{1+\upsilon_2}.$$

We determine the nature of the obtained extremum of the objective function. To do this, take the second derivative of the function:

$$C_{\text{II}}^* = \frac{1}{f_{1\max}^{\frac{m-n}{n}} d_1^{-\frac{k}{m}}} + \frac{1}{f_{2\max}^{\frac{m-n}{m}} \left(d-d_1\right)^{-\frac{k}{m}}}.$$

We get:

$$\frac{\partial^2 C_{\text{II}}^*}{\partial d_1^2} = \frac{\frac{k}{m}\left(\frac{k}{m}-1\right)}{f_{1\max}^{\frac{m-n}{m}} d_1^{z-\frac{k}{m}}} + \frac{\frac{k}{m}\left(\frac{k}{m}-1\right)}{f_{2\max}^{\frac{m-n}{m}} \left(u-d_1\right)^{z-\frac{k}{m}}}.$$

From this equation it is seen that, if $\frac{k}{m}\left(\frac{k}{m}-1\right) < 0$, then the stationary point is the maximum point. In the theory of metal cutting, it is argued that $k < m$, Thus, the obtained solution to the problem of the optimal distribution of the allowance leads to a maximum cost price.

Since the objective function is unimodal, we will look for a solution to the problem at the boundary of the region of admissible values. Consider two alternatives: $d_1 = d_{pm}$, $d_2 = u - d_{pm}$, $d_1 = u - d_{pm}$, $d_2 = d_{pm}$. The first option is preferable if inequality:

$$\frac{1}{f_{1\max}^{\frac{m-n}{n}} d_{pm}^{-\frac{k}{m}}} + \frac{1}{f_{2\max}^{\frac{m-n}{m}} \left(u-d_{pm}\right)^{-\frac{k}{m}}} - \frac{1}{f_{1\max}^{\frac{m-n}{m}} \left(u-d_{pm}\right)^{-\frac{k}{m}}} - \frac{1}{f_{2\max}^{\frac{m-n}{n}} d_{pm}^{-\frac{k}{m}}} < 0.$$

After the transformations we get:

$$\left(f_{1\max}^{\frac{m-n}{m}} - f_{2\max}^{\frac{m-n}{m}}\right)\left(d_{pm}^{-\frac{k}{m}} \left(u-d_{pm}\right)^{\frac{k}{m}}\right) < 0.$$

Since $f_{1\max} > f_{2\max}$, $m > n$, $k > 0$, $m > 0$, $d_{pm} > (u - d_{pm})$, then the resulting inequality always holds. Therefore, underactive type constraints $f_1 < f_{1\max}$, $f_2 \le f_{2\max}$ it is necessary to assign the maximum possible cutting depth on the first pass.

Consider the option when the first pass is actively constraint by force, and on the second – $f \le f_{\max}$ type constraints. We get:

$$C_{\text{II}}^* = \frac{a + be^m}{e^{\frac{\alpha(n-m)}{m\,x}} P_{z01}^{\frac{m-n}{m\,x}} d_1^z} + \frac{\left(a^{m-1} b (m-1)^{1-m}\right)^{\frac{1}{m}} m}{f_{2\max}^{1-\frac{n}{m}} d_2^{-\frac{k}{m}}} = \frac{\varepsilon_1}{P_{z01}^{\frac{m-n}{m\,x}} d_1^z} + \frac{\varepsilon_2 m}{f_{2\max}^{1-\frac{n}{m}} d_2^{-\frac{k}{m}}};$$

$$F = C_{\text{II}}^* + \lambda(u - d_1 - d_2);$$

$$\begin{cases} \dfrac{\partial P}{\partial d_1} = \dfrac{-\varepsilon_1 z}{P_{z01}^{\frac{m-n}{m x}} d_1^{z+1}} - \lambda = 0 \\[2ex] \dfrac{\partial P}{\partial d_2} = \dfrac{\varepsilon_2 k}{f_{2\max}^{1-\frac{n}{m}} d_2^{-\frac{k}{m}+1}} - \lambda = 0 \\[2ex] \dfrac{\partial P}{\partial \lambda} = u - d_1 - d_2 = 0 \end{cases} \quad \begin{array}{l} \rightarrow \quad d_1 = \left(\dfrac{-\varepsilon_1 z f_{2\max}^{1-\frac{n}{m}}}{k\varepsilon_2 P^{\frac{m-n}{m x}}} \right)^{\frac{1}{z+1}} d_2^{\frac{m-k}{m(z+1)}} = \upsilon_3 d_2^{\frac{m-k}{m(z+1)}}; \\ \qquad\qquad \downarrow \\ \rightarrow \quad u - \upsilon_3 d_2^{\frac{m-k}{m(z+1)}} - d_2 = 0; \end{array}$$

$$\upsilon = \left(\frac{-\varepsilon_1 \, z \, f_{2\max}^{1-\frac{n}{m}}}{\varepsilon_2 k P_{z01}^{\frac{m-n}{m x}}} \right)^{\frac{1}{z+1}}.$$

The obtained equation can be solved numerically using standard programs for solving equations of the $f(\mathrm{x}) = 0$ type [38].

Check whether the found stationary point is the minimum of the objective function. Take the second derivative of the function:

$$C_{\mathrm{II}}^{*} = \frac{a + b\ell_1^m}{\ell_1^{\frac{\alpha(n-m)}{m x}} P_{z01}^{\frac{m-n}{m x}} d_1^z} + \frac{\left(a^{m+1} b (m-1)^{1-m}\right)^{\frac{1}{m}} m}{f_{2\max}^{\frac{m-n}{m}} (u - d_1)^{-\frac{k}{m}}}.$$

We get:

$$\frac{\partial^2 C_{\mathrm{II}}^{*}}{\partial d_1^2} = \frac{a + b\ell_1^m}{\ell_1^{\frac{\alpha_1(n-m)}{m x}} P_{z01}^{\frac{m-n}{m x}}} \frac{z(z+1)}{d_1^{z+2}} + \frac{m\left(a^{m-1} b(m-1)^{1-m}\right)^{\frac{1}{m}}}{f_{2\max}^{\frac{m-n}{m}}} \frac{\frac{k}{m}\left(\frac{k}{m} - 1\right)}{(u - d_1)^{\frac{k}{m}+2}} =$$

$$= \varepsilon_1 \frac{z(z+1)}{d_1^{z+2}} + \varepsilon_2 \frac{\frac{k}{m}\left(\frac{k}{m} - 1\right)}{(u - d_1)^{-\frac{k}{m}+2}}.$$

Since $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\dfrac{k}{m} < 1$, then at $z(z+1) < 0$ the stationary point is the maximum. If $z(z+1) > 0$, required evaluate the difference between the first and second terms. If the difference is positive, the stationary point is the minimum point. If the difference is negative, then the stationary point is the maximum point. It can be assumed that in this case, the maximum cutting depth in the first pass will be the maximum permissible value.

Remember that two-pass processing can be more economical than single-pass processing [51-55]. Such conditions may arise, for example, when it is desired to obtain a surface with a low degree of roughness. With a large depth of cutting, this can lead to a sufficiently small value of the $\{V \cdot f\}$ product, which will cause an increase in cost. Using processing in two passes, you can significantly reduce the cost price. It is stated in [6] that processing in two passes can be more economical than processing in one pass, when the machine has a low drive power of the main movement.

Consider the degree of influence of easing constraints on cost price reduction. Let the cutting force constraint be active. Then the cost price of the passage when turning a cylindrical surface is determined from the expression

$$C = \frac{a + b\ell_1^m}{P_{z0}^{\frac{m-n}{m\,x}} \ell_1^{\frac{\alpha_1 n-m}{m\,x}} u^z}.$$

With an increase P_{z0} in k times, the cost price will decrease by a $k^{\frac{m-n}{m\,x}}$ factor. With a decrease in cutting depth in k times, the cost price decreased in k^{-z} times (since $z < 0$ for real values of exponents). For example, when $m = 5$; $n = 1.75$; $k = 0.8$; $\alpha_1 = -0.15$; $\alpha_2 = 0.75$; $\alpha_3 = 1$; $x = 0.81$; $z = -0.9$ we get the following relationships:

1) with an increase in the permissive cutting force from 2000 to 2600 N, the cost of processing will decrease by 1.2 times;

2) with an increase from 2000 to 3000 N, the cost price of machining will decrease by 1.38 times;

3) with reducing the cutting depth from 10 mm to 8 mm, the cost price will decrease by 1.25 times;

4) with a decrease in cutting depth from 10 to 5 mm, the machining cost price will decrease by 1.99 times.

The analytical condition for the "profitability" of two-pass machining instead of a single-pass with active power constraints has the form:

$$\frac{\pi D\ell}{1000}\left(\frac{a + b\ell_1^m}{P_{z01}^{\frac{m-n}{m\,x}} \ell_1^{\frac{\alpha_1(n-m)}{m\,x}} d_1^z} + \frac{a + b\ell_1^m}{P_{z02}^{\frac{m-n}{m\,x}} \ell_1^{\frac{\alpha_1(n-m)}{m\,x}} d_2^z}\right) + a\,t'_m < \frac{\pi D\ell}{1000}\left(\frac{a + b\ell_1^m}{P_{z0}^{\frac{m-n}{m\,x}} \ell_1^{\frac{\alpha_1(n-m)}{m\,x}} d_2}\right);$$

$$\frac{1}{P_{z0}^{\frac{m-n}{m\,x}} u^z} - \frac{1}{P_{z01}^{\frac{m-n}{m\,x}} d_1^z} - \frac{1}{P_{z02}^{\frac{m-n}{m\,x}} d_2^z} > \frac{a d \ell_1^{\frac{\alpha_1(n-m)}{m\,x}} 1000}{(a + b\ell_1^m)\pi D\ell}.$$

Example 4. It is necessary to turn a smooth shaft D = 100 mm, l = 1000 mm, machining allowance u = 10 mm. Let inequality $P_z \le 2000$ N be the active constraint. If the machining is divided into two passes, then the inequality $P_{z1} \le 3200$ N will be the active constraint on the first pass and $P_{z2} \le 2000$ N on the second. The auxiliary time associated with the second transition is t_a = 0.5 min.

Other parameters: a = 1; $b = 9.18 \cdot 10^{-13}$; m = 2.5; n = 0.5; k = 0.3; α_1 = -0.12; α_2= 0.55; α_3 = 0.8; C_p = 310; k_p = 1.

Then

$$x = \frac{0.55 \cdot 2.5 + 0.12 \cdot 0.5}{2.5} = 0.57; \quad \frac{m-n}{m\,x} = \frac{2}{2.5 \cdot 0.57} = 1.40;$$

$$y = \frac{-0.12 \cdot 0.3 - 0.8 \cdot 2.5}{2.5} = 0.81; \quad z = \frac{-0.81 \cdot 2 - 0.3 \cdot 0.57}{2.5 \cdot 0.57} = -1.26;$$

$$(z+1)z > 0; \quad b\ell_1^m = \frac{a(0.55 + 0.12)}{0.55(2.5-1) + 0.12(0.5-1)} = 0.88;$$

$$\ell_1^{\frac{\alpha_1(\mathrm{n-m})}{\mathrm{m\,x}}} = \ell_1^{\mathrm{m}\left(\frac{\alpha_1(\mathrm{n-m})}{\mathrm{m^2\,x}}\right)} = \ell_1^{0.17} = \left(\frac{0.88}{6}\right)^{\frac{0.17}{2.5}} = \left(95.86 \cdot 10^{10}\right)^{0.068} =$$

$$= 1.36 \cdot 4.79 = 6.51;$$

$$\frac{a\,d\,b\,\ell_1^{\frac{\alpha_1(n-m)}{m\,x}} \cdot 1000}{(1 + 0,88) \cdot 3.14 \cdot 100 \cdot 1000} = 0.0055; \quad \upsilon = 0.67^{-5.38} = 8.62;$$

$$d_2 = \frac{10}{1 + 8.62} = 1.03 \approx 1; \quad d_1 = 10 - 1 = 9;$$

$$\frac{1}{2000^{1.4} \cdot 10^{-1.86}} - \frac{1}{3200^{1.4} \cdot 9^{-1.86}} - \frac{1}{2000^{1.4} \cdot 1^{-1.86}} = 0.0049 < 0.0055.$$

Conclusion: for given initial data, two-pass machining is more economical than single-pass processing.

Consider the case when switching to two-pass machining, force constraints do not change. In this case, the necessary condition for greater profitability of two-pass processing is inequality:

$$\frac{a+b\ell_1^m}{P_{z0}^{\frac{m-n}{m\,x}}\,\ell_1^{\frac{\alpha_1(n-m)}{m\,x}}\,u^z} > 2\frac{(a+b\ell_1^m)\,2^z}{P_{z0}^{\frac{m-n}{m\,x}}\,\ell_1^{\frac{\alpha_1(n-m)}{m\,x}}\,u^z},$$

those: $1 > 2^{z+1}$.

Since $z<0$, the last inequality is possible only with $|z|>1$. In example 4, this condition is satisfied. If D = 300 mm, l = 1500 mm, then when partitioning the allowance in half without changing the permissible values of the cutting force, two-pass machining is more economical than single-pass machining.

It must be remembered that the total cost price of the two-pass machining is affected by the accuracy of the size obtained in the first pass. We determine the optimal value of the error in processing the intermediate dimension δ_i.

It is known that the machining error caused by fluctuations in the depth of cut due to the tolerance on the initial size is 0.1 ... 0.2 of the total machining error [7]. Then you can write:

$$wC_{py}k_{py}V^{\alpha_1}f^{\alpha_2}\left[\left(u+\delta_i\right)^{\alpha_3}-u^{\alpha_3}\right]=\frac{P_y}{u^{\alpha_3}}\left[\left(u+\delta_{in}\right)^{\alpha_3}-u^{\alpha_3}\right]=\varepsilon\delta_f,$$

where $\varepsilon=0{,}1...0{,}2$; δ_{in} – error of the initial size, μm; δ_f – error of the final size, μm; w – system compliance, mm/N.

Since then $\delta_{in} \ll u$, :

$$\left(u+\delta_{in}\right)^{\alpha_3}-u^{\alpha_3}\approx\alpha_3\,u^{\alpha_3-1}\,\delta_{in}\rightarrow P_y=\frac{\varepsilon\delta_f u}{\alpha_3\delta_{in}}.$$

For two-pass machining, the cost price is estimated by the dependence

$$C_{II}^{*}=\frac{a+b\ell_1^m}{P_{y01}^{\mu}\,d_1^z}+\frac{a+b\ell_1^m}{P_{y02}^{\mu}\,d_2^z}=\frac{a+b\ell_1^m}{\left(\dfrac{\varepsilon\delta_i d_1}{\alpha_3\delta_{in}C_{py}k_{py}}\right)^{\mu}d_1^z}+\frac{a+b\ell_1^m}{\left(\dfrac{\varepsilon\delta_f d_2}{\alpha_3\delta_i\,C_{py}k_{py}}\right)^{\mu}d_2^z};$$

$$\mu=\frac{\mathrm{m-n}}{\mathrm{m\,x}}>0;$$

$$\frac{\partial C_{II}^{*}}{\partial\delta_i}=\frac{a+b\ell_1^m}{\left(\dfrac{\varepsilon}{\alpha_3C_{py}k_{py}}\right)^{\mu}}\left(\frac{-\delta_{in}^{\mu}\mu}{\delta_i^{\mu+1}\,d_1^{z+\mu}}+\frac{\mu\,\delta_i^{\mu-1}}{\delta_f^{\mu}\,d_2^{z+\mu}}\right)=0;$$

$$\delta_i^{2\mu}=\left(\frac{d_2}{d_1}\right)^{z+\mu}\delta_{in}^{\mu}\delta_f^{\mu}\rightarrow\delta_i=\sqrt{\left(\frac{d_2}{d_1}\right)^{\frac{z}{\mu}+1}}\,\delta_{in}\delta_f\,.$$

We determine the nature of the obtained stationary point:

$$\frac{\partial^2C_{II}^{*}}{\partial\delta_i^2}=\frac{a+b\ell_1^m}{\left(\dfrac{\varepsilon}{\alpha_3C_{py}k_{py}}\right)^{\mu}}\left(\frac{\mu(\mu+1)\delta_{in}^{\mu}}{\delta_i^{\mu+z}\,d_1^{z+\mu}}+\frac{\mu(\mu-1)\,\delta_i^{\mu-z}}{\delta_f^{\mu}\,d_2^{z+\mu}}\right)=\frac{\left(a+b\ell_1^m\right)\mu}{\left(\dfrac{\varepsilon}{\alpha_3C_{py}k_{py}}\right)^{\mu}\delta_i^2}\cdot$$

$$\left(\frac{(\mu+1)\delta_i^{\mu}}{\sqrt{\left(\dfrac{d_2}{d_1}\right)^{z+\mu}}\,\delta_{in}^{\mu}\delta_f^{\mu}\,d_1^{z+\mu}}+\frac{(\mu-1)\sqrt{\left(\dfrac{d_2}{d_1}\right)^{z+\mu}}\,\delta_{in}^{\mu}\delta_f^{\mu}}{\delta_f^{\mu}d_2^{z+\mu}}\right)=$$

$$=\frac{\left(a+b\ell_1^m\right)\mu}{\left(\dfrac{\varepsilon}{\alpha_3C_{py}k_{py}}\right)^{\mu}\delta_i^2}\left(\frac{\delta_{in}^{\frac{\mu}{z}}(\mu+1)}{d_2^{\frac{z+\mu}{2}}\,d_1^{\frac{z+\mu}{2}}\,\delta_f^{\frac{\mu}{2}}}+\frac{(\mu-1)\delta_{in}^{\frac{\mu}{z}}}{d_2^{\frac{z+\mu}{2}}\,d_1^{\frac{z+\mu}{2}}\,\delta_f^{\frac{\mu}{2}}}\right)>0.$$

Thus, the stationary point is the minimum point. This allows you to calculate the value of the intermediate machining error ensuring the minimum cost price of two-pass processing.

3.4. Three-pass optimization

We analyze the three-pass processing [31]. Consider the case when force constraint is active on each pass. Then, by analogy with two-pass machining, the mathematical model of the problem can be represented in the following form:

$$\begin{cases} C^*_{\text{III}} = \dfrac{a+b\ell_1^m}{P_{z01}^{\frac{m-n}{m\,x}}\ell_1^{\frac{\alpha_1(n-m)}{m\,x}}d_1^z} + \dfrac{a+b\ell_1^m}{P_{z02}^{\frac{m-n}{m\,x}}\ell_1^{\frac{\alpha_1(n-m)}{m\,x}}d_2^z} + \dfrac{a+b\ell_1^m}{P_{z03}^{\frac{m-n}{m\,x}}\ell_1^{\frac{\alpha_1(n-m)}{m\,x}}d_3^z} \to \min; \\ u-d_1-d_2-d_3=0. \end{cases}$$

Using the Lagrange multiplier method, we can show that the stationary point is determined by the relation

$$\frac{1}{P_{z01}^{\frac{m-n}{m\,x}}d_1^{z+1}} = \frac{1}{P_{z02}^{\frac{m-n}{m\,x}}d_2^{z+1}} = \frac{1}{P_{z03}^{\frac{m-n}{m\,x}}d_3^{z+1}}.$$

Then the following dependencies take place:

$$d_2 = \chi_1 d_1 = \left(\frac{P_{z01}}{P_{z02}}\right)^{\frac{m-n}{m\,x(z+1)}} d_1; \quad d_3 = \chi_2 d_1 = \left(\frac{P_{z01}}{P_{z03}}\right)^{\frac{m-n}{m\,x(z+1)}} d_1;$$

$$u - d_1 - \chi_1 d_1 - \chi_2 d_1 = 0 \to d_1 = \frac{u}{1+\chi_1+\chi_2};$$

$$d_2 = \frac{\chi_1 u}{1+\chi_1+\chi_2}; \qquad d_3 = \frac{\chi_2 u}{1+\chi_1+\chi_2},$$

where $\chi_1 = \left(\dfrac{P_{z01}}{P_{z02}}\right)^{\frac{m-n}{m\,x(z+1)}}$; $\chi_2 = \left(\dfrac{P_{z01}}{P_{z03}}\right)^{\frac{m-n}{m\,x(z+1)}}$.

We study the nature of the stationary point. To understand whether a stationary point affords a relative minimum of the objective function, we need to find out the behavior of the quadratic form

$$\sum_{i,\,j=1}^{n} \frac{\partial^2 F}{\partial x^i \partial x^j} \; (\hat{\text{x}},\ \lambda)\,\xi^i \zeta^j,$$

for vectors ξ satisfying the equalities

$$\sum_{i=1}^{n} \frac{\partial g_k(\hat{\text{x}})}{\partial x^i} = 0; \quad k = 1, 2, ..., m,$$

where g_k k_{th} – the constraint of the form: $g_k(\overline{x}) = 0$.

In this case, it is necessary to find out the sign-definiteness of the quadratic form [12]:

$$\frac{z(z+1)}{P_{z01}^{\frac{m-n}{m\,x}}d_1^{z+2}}(\xi)^2 + \frac{z(z+1)}{P_{z02}^{\frac{m-n}{m\,x}}d_2^{z+2}}(\xi)^2 + \frac{z(z+1)}{P_{z03}^{\frac{m-n}{m\,x}}d_3^{z+2}}(\xi)^2,$$

given that $\xi^1 + \xi^2 + \xi^3 = 0$.

It is quite obvious that $z(z+1)>0$ a stationary point is a minimum. When $z(z+1)<0$ the stationary point is the maximum and the solution must be sought at the boundary of the region of admissible values.

We find the minimum point at the boundary of the region of the admissible value. Let the equalities $d_1 = d_{pm}$; $d_2 = d_{pm}$; $d_3 = u - 2d_{pm}$ define the first alternative option, and the equalities $d_1 = (u - 2d_{pm})$; $d_2 = d_{pm}$; $d_3 = d_{pm}$ – the second option. It can be argued that $P_{1\,pm} \geq P_{2\,pm} > P_{3\,pm}$. Then the first option is more economical than the second, provided that inequality is satisfied:

$$\frac{1}{P_{z01}^{\frac{m-n}{m x}} d_{pm}^{z}} + \frac{1}{P_{z02}^{\frac{m-n}{m x}} d_{pm}^{z}} + \frac{1}{P_{z03}^{\frac{m-n}{m x}} \left(u - 2d_{pm}\right)^{z}} - \frac{1}{P_{z01}^{\frac{m-n}{m x}} \left(u - 2d_{pm}\right)} -$$
$$- \frac{1}{P_{z02}^{\frac{m-n}{m x}} d_{pm}^{z}} - \frac{1}{P_{z03}^{\frac{m-n}{m x}} d_{pm}^{z}} < 0.$$

After performing a series of transformations, we obtain:

$$\left(u - 2d_{pm}\right)^{z} \left(P_3^{\frac{m-n}{m x}} P_2^{\frac{m-n}{m x}} - P_1^{\frac{m-n}{m x}} P_2^{\frac{m-n}{m x}} \right) - d_{pm}^{z} \left(P_2^{\frac{m-n}{m x}} P_3^{\frac{m-n}{m x}} - P_1^{\frac{m-n}{m x}} P_2^{\frac{m-n}{m x}} \right) < 0.$$

Because $\left(P_3^{\frac{m-n}{m x}} P_2^{\frac{m-n}{m x}} - P_1^{\frac{m-n}{m x}} P_2^{\frac{m-n}{m x}} \right) < 0;\ (u - 2d_{pm}) < d_{pm};\ z < 0,$

then inequality always holds.

If the power constraints are equal, both options have the same cost price. If the force constraints are equal at each pass and the stationary

point is the minimum point, the optimal distribution of the allowance between the passes is determined by equality $d_1 = d_2 = d_3$. .

Provided that type constraints $f_i \leq f_{i\,\max}$, are active, the distribution of the allowance for three-pass processing is determined by the equalities

$$d_2 = \chi_1^{*} d_1;\quad d_3 = \chi_2^{*} d_1;\quad \chi_1^{*} = \left(\frac{f_{1\,\max}}{f_{2\,\max}}\right)^{\frac{m-n}{m}} \quad \chi_2^{*} = \left(\frac{f_{1\,\max}}{f_{2\,\max}}\right)^{\frac{m-n}{m}}.$$

Having performed the analysis of the stationary point on the nature of the extremum and using the quadratic form of the Lagrange function for this, we can show that it is always the maximum. Therefore, the minimum cost price should be sought at the marginal distribution of allowance. Similarly, we obtain the inequality:

$$\left(u - 2d_{pm}\right)^{-\frac{k}{m}} \left(f_{3\,\max}^{\frac{m-n}{m x}} f_{2\,\max}^{\frac{m-n}{m x}} - f_{1\,\max}^{\frac{m-n}{m x}} f_{2\,\max}^{\frac{m-n}{m x}} \right) -$$
$$- d_{pm}^{-\frac{k}{m}} \left(f_{3\,\max}^{\frac{m-n}{m x}} f_{2\,\max}^{\frac{m-n}{m x}} - f_{1\,\max}^{\frac{m-n}{m x}} f_{2\,\max}^{\frac{m-n}{m x}} \right) < 0,$$

which is always satisfied subject to $f_{1\,\max} \geq f_{2\max} > f_{3\,\max}$. If there is equality $f_{1\,\max} = f_{2\max} = f_{3\,\max}$, both marginal variants of the distribution of allowance have the same cost.

3.5. Optimization of cutting conditions during finishing turning

The main difficulty in optimizing the cutting modes when finishing turning external surfaces is that the cutting speed is often a variable, depending on the diameter of the cross-section of the workpiece at its point of contact with the cutting tool. This greatly complicated the use of optimization methods in practice, especially for CNC machines [56-58]. Therefore, it is necessary to develop a mathematical model that allows analytically calculating the optimal values of cutting modes, taking into account the machining of the workpiece.

Consider the case when in one instrumental step it is required to turning two shaft steps with parameters D_1, l_1 and D_2, l_2. The frequency of rotation n, depth of cut d and feed f during the processing of both steps remain unchanged. We find the value of the equivalent cutting speed V, which provides the same tool life as the cutting speed V_1 and V_2.

For further discussion, we accept the following initial premises.

1. Tool life T: $T = \dfrac{C_T}{V^m f^n d^k}$,

where C_T – constant for specific machining conditions.

2. The amount of wear h of the tool, that linearly depends on the machine time of its operation t_m within the limits of tool life T:

$$h = kt_m = \frac{h_{pm}}{T} t_m = \frac{h_{pm} V^m f^n d^k}{C_T} t_m,$$

where h_{pm} – permissible wear of the tool.

3. Permissible tool wear – constant value for various values of cutting conditions. Then you can write the following expression:

$$h_\Sigma = h_1 + h_2 = k_1 t_{m1} + k_2 t_{m2} = k_e (t_{m1} + t_{m2}),$$

where h_Σ – the total wear for the entire instrumental step; h_1, h_2 – tool wear after machining, respectively, the first and second stages; k_e – proportionality coefficient at equivalent cutting speed V_e.

Next, we have:

$$\frac{t_{m1}}{t_{m1} + t_{m2}} V_1^m + \frac{t_{m2}}{t_{m1} + t_{m2}} V_2^m = V_e^m.$$

When rotation frequency n we get the expression for the equivalent diameter:

$$D_e^m = \frac{t_{m1}}{t_{m1} + t_{p2}} D_1^m + \frac{t_{m2}}{t_{m1} + t_{m2}} D_2^m.$$

The resulting expression is easily generalized to the case of n_s stages:

$$D_э^m = \frac{t_{m1}}{\sum^n t_{mj}} D_1^m + \frac{t_{m2}}{\sum^n t_{mj}} D_2^m + \ldots + \frac{t_{mn_s}}{\sum^n t_{mj}} D_{n_s}^m.$$

At feed f = const:

$$D_e^m = \frac{l_{1v}}{\sum^{n_s} l_i} D_1^m + \frac{l_{2v}}{\sum^{n_s} l_i} D_2^m + \ldots + \frac{l_{nv}}{\sum^{n_s} l_i} D_{n_s}^m.$$

Consider the treatment of a conical surface, presented in the form of a set of cylinders with height Δx and initial (min) diameter D_{in} (fig. 3.2).

Then the desired equivalent diameter D_e at constant rotation frequency n and feed rate f takes value:

$$D_e^m = \frac{\Delta x}{l} D_{in}^m + \frac{\Delta x}{l}(D_{in} + \Delta x\, tg\alpha)^m + \frac{\Delta x}{l}(D_{in} + 2\Delta x\, tg\alpha)^m + \ldots$$
$$+ \frac{\Delta x}{l}(D_{in} + (n_s - 1)\Delta x\, tg\alpha)^m.$$

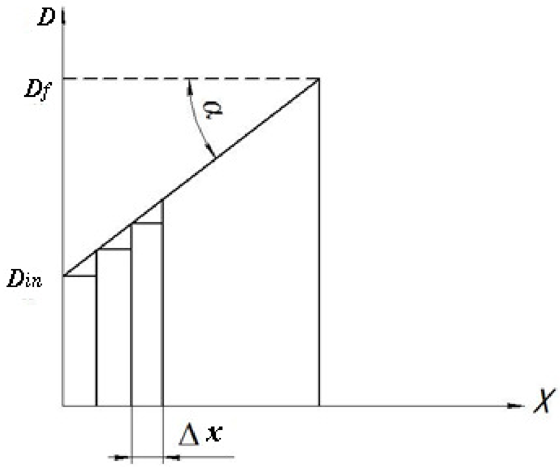

Fig. 3.2. Conical surface treatment

Passing to the limit, we obtain:

$$D_e^m = \frac{1}{l}\int_0^l \left(D_{in} + x tg\alpha\right)^m \alpha_x = \frac{\left(D_{in} + l \cdot tg\alpha\right)^{m+1} - D_{in}^{m+1}}{l \cdot tg\alpha(m+1)}.$$

Consider the machining of the end surface. We represent it in the form of a set n of rings wide Δx (fig. 3.3).

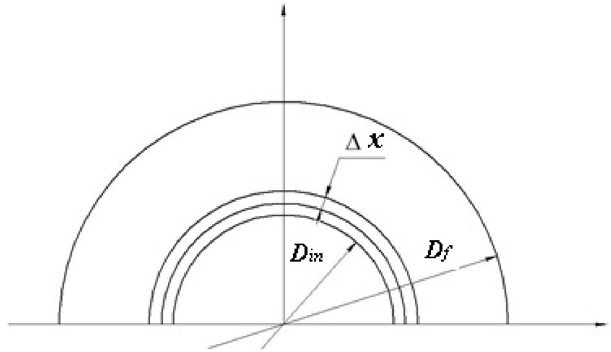

Fig. 3.3. End surface treatment

If Δx is small enough, it can be considered that the turning of a ring with a smaller diameter is equivalent to the turning of a cylinder with the same diameter in length Δx. Then the equivalent diameter

$$D_e^m = \frac{\Delta x}{D_f - D_{in}} D_{in}^m + \frac{\Delta x}{D_f - D_{in}}(D_{in} + \Delta x)^m + \ldots + \frac{\Delta x}{D_f - D_{in}}(D_{in} + (n_s - 1)\Delta x)^m.$$

Passing to the limit, we obtain

$$D_e^m = \frac{1}{D_f - D_{in}} \int_0^{D_f - D_{in}} \left(D_{in} + x\right)^m d\,x = \frac{D_f^{m+1} - D_{in}^{m+1}}{(m+1)(D_f - D_{in})}.$$

If the workpiece profile on a length segment l is described by a function $f(x)$, then the equivalent diameter D_e^m determined by the following relationship:

$$D_e^m = \frac{1}{l}\int_0^l f^m(x)dx.$$

The results obtained allow us to solve the problems of optimizing cutting modes during finishing in an analytical form. For example, for two-stage processing, the objective function can be represented as follows:

$$C = \frac{a + bV_e^m f^n d^k}{V_e f} = \frac{a + b\left(\frac{\pi D_e n}{1000}\right)^m f^n d^k}{\frac{\pi D_t n}{1000} f} \to \min,$$

where *a, b* – constant coefficients; *C* – the proportion of the cost price of the step, depending on the cutting modes;

$$D_e' = \left(\frac{l_{1v}}{l_{1v} + l_{2v}} D_1^m + \frac{l_2}{l_{1v} + l_{2v}} D_2^m\right)^{\frac{1}{m}}.$$

Then, with a restriction on surface roughness $R_z \le R_{z\,pm}$; $R_z = C_R n^{\alpha_1} f^{\alpha_2} d^{\alpha_3}$, one can find the optimal values of the rotational speed and feed rate that minimize the objective function:

$$n_0 = \left(\left(\frac{R_{z\,pm}}{C_R}\right)^{-n} l^{m\,x + \alpha_1 n} d^{-(kx - n\,y)}\right)^{\frac{1}{m\,x}};\quad f_0 = \left(\left(\frac{R_{z\,pm}}{C_R}\right) l^{-\alpha_1} d^{y}\right)^{\frac{1}{x}};$$

$$l^m = \frac{a(\alpha_1 - \alpha_2)}{b\left(\frac{\pi D_e}{1000}\right)^m \left(\alpha_2(m-1) - \alpha_1(n-1)\right)};\quad x = \frac{\alpha_2 m - \alpha_1 n}{m};\quad y = \frac{\alpha_1 n - \alpha_3 m}{m}.$$

Besides, it became possible to analytically solve the problem of optimizing cutting modes during two-pass machining with one tool.

Let it be required to turning the shaft journal in two passes, while the rotational speed and feed remain constant. Strength constraints P_1, P_2 are set, respectively, on the first and second pass:

$$P_1 = C_p n^{\alpha_1} f^{\alpha_2} d_1^{\alpha_3};\quad P_2 = C_p n^{\alpha_1} f^{\alpha_2} d_2^{\alpha_3}.$$

From these constraints, it follows that the depth of cut:

$$d_1 = \frac{u}{1+\upsilon};\quad d_2 = \frac{\upsilon u}{1+\upsilon};\quad \upsilon = \left(\frac{P_1}{P_2}\right)^{-\frac{1}{\alpha_3}};\quad u = d_1 + d_2,$$

where *u* – total processing pass.

We write down the objective function as a part of the piece time t_{pc}, depending on the cutting modes:

$$t_{pc} = \left(t_c + t_c \frac{t_{ch}}{T}\right) = 2l\left(\frac{1 + b n^m f^n d_e^k}{n f}\right) = 2l\left(\frac{1 + b n^m f^n \left(\frac{1}{2} d_1^k + \frac{1}{2} d_2^k\right)}{n f}\right),$$

where t_{ch} – tool change time; d_e – equivalent cutting depth; *b* – constant value.

From $\frac{d\,t_{pc}}{d\,n} = 0$ we get:

$$n = \left(\frac{2}{bf^n (m-1)(d_1^k + d_2^k)}\right)^{\frac{1}{m}}.$$

From force or other constraints in the form of power functions we have:

$$f_1 = \left(\frac{P_1}{d_1^{\alpha_3} C_p} \right)^{\frac{1}{\alpha_2}} n^{-\frac{\alpha_1}{\alpha_2} n}; f_2 = \left(\frac{P_2}{d_2^{\alpha_3} C_p} \right)^{\frac{1}{\alpha_2}} n^{-\frac{\alpha_1}{\alpha_2} n} = f.$$

Thus, the obtained dependences for determining equivalent cutting modes made it possible to analytically solve practical problems in optimizing the parameters of machine operations.

3.6. Optimization of cutting modes using the sequential method accounting constraints

Part of the machining detail cost price with a cutting tool in one pass, depending on the values of the cutting and feed speeds equal to:

$$C = A \cdot t_m + A \cdot t_{ch} \frac{t_c}{T} + B \frac{t_c}{T}, \tag{3.8}$$

where A – total cost price of one minute of the machine and worker, cent/min; $t_м$ - machine time, min; t_{ch} – tool change time, min; t_c – cutting time, min; T – tool life, min; B – the cost of the tool transferred to the detail for one period of durability, cent.

To solve our problem with sufficient reason, we can write:

$$t_m = t_c = \frac{\pi Dl}{1000Vf}. \tag{3.9}$$

We represent the tool life equation for the turning operation in the following form:

$$T = \frac{K}{V^{n_1} f^{n_2}}, \tag{3.10}$$

Where $K = \frac{C_v}{d^{x_v}}$.

According to the condition of the task, the cutting depth is constant and is selected from the table data.

Substituting equations (3.9) and (3.10) into equation (3.8), we have:

$$\begin{aligned} C &= A\frac{\pi Dl}{1000Vf} + At_{ch}\frac{\pi Dl}{1000K} V^{n_1-1} f^{n_2-1} + B\frac{\pi Dl}{1000K} V^{n_1-1} f^{n_2-1} = \\ &= XV^{-1} f^{=1} + YV^{n_1-1} f^{n_2-1} + ZV^{n_1-1} f^{n_2-1}, \end{aligned} \tag{3.11}$$

where $X = A\frac{\pi Dl}{1000}$; $Y = A\frac{\pi Dl}{1000}\frac{t_{ch}}{K}$; $Z = Б\frac{\pi Dl}{1000}\cdot\frac{1}{K}$.

A necessary condition for the minimum cost price is the zero gradients of the function (3.11), i.e. all partial derivatives:

$$\frac{\partial C}{\partial V} = 0; \qquad \frac{\partial C}{\partial f} = 0.$$

From these equations it follows:

$$\begin{aligned} \frac{\partial C}{\partial V} &= -X\frac{1}{fV^2} + Yf^{n_2-1}(n_1-1)V^{n_1-2} + Zf^{n_2-1}(n_1-1)V^{n_1-2} = \\ &= 0 \to -X + Y(n_1-1)V^{n_1} f^{n_2} + Z(n_1-1)V^{n_1} f^{n_2} = 0; \end{aligned} \tag{3.12}$$

$$\begin{aligned} \frac{\partial C}{\partial f} &= -X\frac{1}{f^2 V} + YV^{n_1-1}(n_2-1)f^{n_2-2} + ZV^{n_1-1}(n_2-1)f^{n_2-2} = \\ &= 0 \to -X + Y(n_2-1)V^{n_1} f^{n_2} + Z(n_2-1)V^{n_1} f^{n_2} = 0. \end{aligned} \tag{3.13}$$

From (3.12) and (3.13) it follows that equality must hold

$$\frac{X}{Y+Z}=(n_1-1)V^{n_1}f^{n_2}=(n_2-1)V^{n_1}f^{n_2}.$$

This equation is solvable only if the exponents are equal for V and f. Since this is practically impossible, function (3.11) does not have a single minimum.

The graphs of functions (3.12) and (3.13) are shown in fig. 3.4.

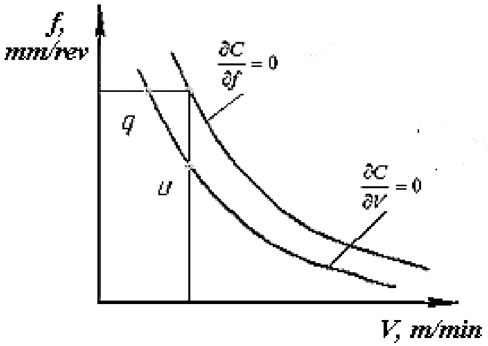

Fig. 3.4. Level lines by the criterion of minimum cost price

The axis on which the machining cost price is laid off is perpendicular to the V-f plane. For a turning point $n_2 < n_1$, therefore, for any feed value f, the cutting speed satisfying equation (3.12) will be lower than the cutting speed calculated from equation (3.13).

In [31], it was shown that the cost price at q is lower than at r, and at r is lower than at u. Therefore, the cost price of the part decreases with increasing feed. The cutting modes corresponding to the permissible minimum cost price of pass should be selected from the maximum

allowable feed and cutting speed calculated according to equation (3.12), which can be represented in the following form:

$$\frac{K}{V^{n_1}f^{n_2}}=(n_1-1)\left(\frac{At_{ch}+B}{A}\right)=T_V,$$

where T_V – tool life corresponding to the minimum cost of the detail.

The optimum speed corresponding to tool life is determined from the expression

$$V=\frac{K^{\frac{1}{n_1}}}{T_V^{\frac{1}{n_1}}f^{\frac{n_2}{n_1}}},$$

where f – maximum possible feed value.

Thus, the procedure for calculating the optimal cutting modes will be as follows:

1. Choose the maximum feed, based on the capabilities of the machine.
2. Calculate the cutting speed V_0 from equation (3.12).
3. The calculated speed is compared with the maximum machine speed V_{max}. If $V_0 < V_{max}$, then no adjustment is required. Otherwise, you should take the maximum speed provided by the machine.
4. We take into account constraints on the maximum capacity of the machine. For turning, the cutting speed allowed by the machine is calculated according to the formulas:

$$V_{mach}=\left(\frac{6120N_e^{mach}K_{ct}}{C_{Pz}d^{x_{pz}}f^{y_{Pz}}K_{Pz}}\right)^{\frac{1}{1+n}};\quad N_e^{mach}=N_{mach}\eta,$$

where K_{ct} = 1.25 when machining with a carbide tool;

K_{ct} = 1.0 when machining a high-speed tool;

N_{mach} – capacity of the drive motor for the main movement of the machine, kW;

$K_{Pz}, C_{Pz}, x_{Pz}, y_{Pz}, n$ – constant, correction factor and exponents in the formula for determining the cutting force P_z:

$$P_z = C_{Pz} d^{x_{pz}} f^{y_{Pz}} V^n K_{Pz}. \qquad (3.14)$$

Taking into account the value of V_{mach}, the cutting speed is adjusted.

5. Check the constraints on the maximum cutting force. For turning, the magnitude of this force is determined by the formula (3.14). If the cutting force is greater than the value allowed by the machine feed mechanism, it is necessary to reduce the feed or choose another machine.

6. We calculate the feed allowed from the condition of ensuring the required roughness. The height of the microroughness can be approximately determined from the ratio:

$$R_z = \frac{f^2}{8r},$$

where r – radius at the tip of the cutter.

Therefore, for given surface roughness and tool geometry, there is a feed limit value. If this value is less than the previously determined feed value, then the feed is assumed to equal $f = \sqrt{8rR_z}$, and the cutting speed is calculated according to equation (3.12).

7. The last constraint is caused by the stepwise nature of the feed and speed changes. The cutting modes are finally determined by finding the maximum allowable feed value according to the machine passport and calculating V_0. Then, such a rotation speed is selected that provides a value of real speed that is as close as possible to the calculated value.

Using the sequential method accounting constraints

Consider the procedure for optimizing cutting modes when turning according to the criterion of minimum cost price.

Initial data: The workpiece is a shaft with a diameter D_1 = 200 mm and a length l = 1000 mm;

Machine model 16K20F3; Limit machining modes: f_{min} = 0.05 mm/rev; f_{max} = 2.8 mm/rev; n_{min} = 12.5 min^{-1}; n_{max} = 2000 rpm. The specified cutting depth d = 3 mm; drive power N_{mach} = 10 kW; η = 0. 65.

Tool change time t_{ch} = 0.08 min.

We will calculate according to the criterion of minimum cost price by the standards given in [30].

1. Choose the maximum feed, based on the capabilities of the machine f = 2.8 mm / rev.

2. Using the formula (3.12), we calculate the cutting speed:

$$V = \sqrt[n_1]{\frac{X}{(Y+X)(n_1-1)f^{n_2}}} = \sqrt[5]{\frac{373}{5.21\cdot10^{-12}+2192\cdot10^{-12}(5-1)2.8^{2,25}}} = 84.0 \text{ m/min.}$$

where

$$X = \frac{A\pi D_1 l}{1000} = \frac{0,594\cdot3.14\cdot200\cdot1000}{1000} = 373;$$

$$Y = X\frac{t_{ch}}{K} = 373\cdot\frac{0.08}{5.73\cdot10^{12}} = 5.21\cdot10^{-12};$$

$$Z = \frac{B\pi D_1 l}{1000}\cdot\frac{1}{K} = \frac{20\cdot3.14\cdot200\cdot1000}{1000}\cdot\frac{1}{5.73\cdot10^{12}} = 2192\cdot10^{-12};$$

$$K = \frac{C_V^{\frac{1}{m}}}{d^{\frac{x}{m}}} = \frac{420^{\frac{1}{0.2}}}{3^{\frac{0.15}{0.2}}} = 5.73\cdot10^{-12},$$

where A = 0.594 cent/min; B = 20 cent; C_V = 420; x = 0.15; y = 0.45; m = 0.2 [5].

3. Compare the resulting speed (m/min) with the permissible machine speeds:

$$V_{\max} = \frac{\pi D_1 n_{\max}}{1000} = \frac{3.14 \cdot 200 \cdot 2000}{1000} = 1256;$$

$$V_{\min} = \frac{\pi D_1 n_{\min}}{1000} = \frac{3.14 \cdot 200 \cdot 12.5}{1000} = 7.85.$$

Since $V_{min} < V < V_{max}$, we take V = 84 m/min.

4. Check the speed limit (m/min) for machine capacity, [9], for this, we calculate the cutting speed according to the formula

$$V_{mach} = \left(\frac{6120 N_e^{ach} K_{ct}}{C_{Pz} d^{x_{Pz}} f^{y_{Pz}} K_z}\right)^{\frac{1}{1+n}} = \left(\frac{6120 \cdot 8,0 \cdot 1,25}{300 \cdot 3^1 \cdot 2,8^{0.75} \cdot 0,84}\right)^{\frac{1}{1-0.15}} = 72,$$

where $N_e^{mach} = 10 \cdot 0.8 = 8.0$ kW.

K_{ct} = 1.25 when machining with carbide cutter [30].

C_{Pz} = 300, x_{Pz} = 1.0; y_{Pz} = 0.75; n = -0.15 [30];

$K_z = K_{MP} \cdot K_{\gamma P} \cdot K_{\varphi P} \cdot K_{\lambda P} \cdot K_{zP}$ = 0.84.

K_{mr} = 0.84 [5]; $K_{MP} = K_{\gamma P} = K_{\varphi P} = K_{\lambda P} = K_{zP}$ = 1 [30].

Since $V > V_{mach}$, we take $V = V_{mach}$ = 72 m/min.

5. We determine the cutting force by the according to the well-known formula:

$$P_z = C_{Pz} d^{x_{Pz}} f^{y_{Pz}} V^n K_z = 300 \cdot 3^1 \cdot 2.8^{0.75} \cdot 72^{-0.15} \cdot 0.84 = 860 \text{ N}.$$

6. Determine tool life (effective):

$$T_V = (n_1 - 1)\left(\frac{A t_{ch} + B}{A}\right) = (5-1)\left(\frac{0.594 \cdot 0.08 + 20}{0.594}\right) = 135 \text{ min}.$$

7. Determine the height of the microroughness:

$$R_z = \frac{f^2}{8r} = \frac{2.8^2}{8 \cdot 1.5} = 0.65,$$

where r = 1.5 – radius at the cutter tip.

8. Determine the machine cost price (cent):

$$C = A \cdot t_м + A \cdot t_{см} \frac{t_p}{T} + B \frac{t_p}{T} = 0.594 \cdot 3.11 + 0.584 \cdot 0.08 \cdot \frac{3.11}{135} + 20 \cdot \frac{3.11}{135} = 2.31,$$

Where $t = t_m = \dfrac{3.14 \cdot 200 \cdot 1000}{1000 \cdot 72 \cdot 2.8} = 3.11$ min.

3.7. Stochastic optimization cutting processes

Most work on optimizing the parameters of the metalworking process assumes that all parameters of the cutting process are deterministic. However, the physical phenomena that accompany the real machining process are largely probabilistic. Therefore, consideration of the main indicators – the tool life and strength of the cutting tool, as determinate values introduces significant errors in the results of determining the optimal parameters of the cutting process. The works [59–61] are known in which the probabilistic formulation of the optimization problem is already implemented. Mathematical models are constructed in which the objective function and constraints are probabilistic.

Typical examples of this are the tool life equations and cutting forces, which are often considered both in the objective function and in the constraints. The degree of tool wear in the cutter batch is different due to characteristic changes in the wear mechanism with varying experimental conditions. Thus, the tool life, determined experimentally by the value of its wear, should be considered as a random rather than deterministic value. The coefficients in the empirical equation for the cutting force should also be considered as random variables. One of the advantages of such models is the possibility of sequentially setting the level of confidence in fulfilling the constraints of the machining process. At the same time, the assumption that all random variables are independent and obey the normal distribution law is typical for most models.

The solution of stochastic problems in the general case requires not systems of numbers, but systems of functions or probability distributions.

For specific machining conditions, the cutting modes determined taking into account the influence of random deviations may differ from the modes found from the averaged data. Replacing random variables with their mathematical expectation over several realizations of random factors is advisable when the range of random variables variation is small, and also when the random factors have a slightly large spread, but the characteristics of the process depend linearly (or almost linearly) on them. To do this, apply techniques that approximate nonlinear dependencies by their linear approximations.

There are two classes of methods for solving the optimization problem in a probabilistic formulation:

- based on the search for the exact or approximate deterministic equivalent of the stochastic problem with the subsequent application of nonlinear programming methods, which are called indirect stochastic programming methods [62];
- allowing to solve the problem of stochastic programming based on the values of random functions.

Methods of the first class are focused on certain laws of distribution (mainly on the normal law of distribution) [61, 62], and their implementation is associated with complex analytical studies. Moreover, the techniques used for some types of distributions are unacceptable for other distributions.

The methods of the second class operate only with the values of random functions $F_i(\omega, x)$, were ω – set of random parameters: $\omega = (\sigma_В, \sigma_{0,2}, \psi, \sigma_и, \mathrm{d}, \mathrm{t})$ including the physicomechanical properties of the material of the workpiece $(\sigma_В, \sigma_0, \sigma_и, \psi)$, uneven allowance and, as a consequence, the variability of the depth of cut; x – vector of variable controlled parameters. As the latter, cutting mode elements and tool geometry parameters are most often used (as in the deterministic setting).

In the general case, if there are both probabilistic and deterministic constraints, the statement of the problem of stochastic optimization can be presented in he following form.

To find $\eta' = \min\{M_\omega \eta_i(x,\omega)\} \quad x \in D_s$,

under constraints $P_\omega\{F_k(x,\omega) \le 0\} \ge [\alpha_k]; \quad F_i(x) \le 0$,

where: $k = 1, 2, ..., k_1; \quad i = k_1, k_2, ..., m.$

Here, the mathematical expectation of the objective function (energy criterion η) is minimized by searching for optimal values of controlled parameters x, appropriate areas of feasible solutions for the selected constraint system $F_i(x,\omega)$. The constraint system includes preset feasible probabilities

$$[\alpha] = [\alpha_1], [\alpha_2], ..., [\alpha_{k1}].$$

To the process of turning on CNC machines, the model of the stochastic optimization problem can be represented as follows:

to find $\eta_1 = \min\{M_\omega \eta_1(f,V) | f, V \in D_s\}$,

under the following constraints:

$$\left.\begin{aligned} F_1(f,V,\omega) &= t_m(f,V) - T(f,V,\omega) \le 0; \\ F_2(f,V,\omega) &= T_{np} - T(f,V,\omega) \le 0; \\ F_3(f,V,\omega) &= P(f,V,\omega) - [P] \le 0; \\ F_4(f,V,\omega) &= R_a(f,V,\omega) - [R_a] \le 0; \\ F_5(f,V,\omega) &= h_r(f,V) - [h_r] \le 0. \end{aligned}\right\}$$

The meaning of the admissible probability of fulfillment of the constraints lies in the probability that the vector $x = \{V, f\}$ (in the above statement) belongs to a certain range of possible: values D_s. The allowable probability $[\alpha]$ can be assigned differentially for each constraint or with one value for all constraints depending on the machining requirements.

In many empirical dependences of cutting theory, there is an indicator of tool life *T*, the mathematical expectation $\left(\overline{\mathrm{T}}\right)$, which can be calculated analytically [63]:

$$\overline{T} = \frac{he^{k_T}}{V},$$

where h – the maximum allowable wear of the cutting tool [63]; V – cutting speed; K_T – indicator representing a second-order polynomial of the form:

$$k_T = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{1 \le i < j \le n} b_{ij} x_i x_j + \sum_{i=1} b_{ii} x_i^2, \qquad (3.15)$$

where b_0, b_i, b_{ij}, b_{ii} – coefficients of the regression model; $x = \{x_i, x_j\}$ – vector of controlled variables. In equation (3.15), the vector $\overline{x} = f\left(V, \overline{\theta}, \overline{H}_d, \overline{H}_t, \overline{P}\right)$, where $\overline{\theta}, \overline{H}_d, \overline{H}_t, \overline{P}$ – mathematical expectations of

the temperature in the cutting zone, the hardness of the processed and tool materials, and the cutting force. In turn, the mathematical expectations of the listed quantities as a function of many parameters of the cutting process are expressed by the polynomial (3.15).

In the practice of technological design, not only formalized statements are used, but also algorithms and programs for solving problems of stochastic optimization of cutting modes [59-61]. To solve these problems, methods are used such as searching for a global extremum, which is based on a combination of statistical test procedures (Monte Carlo procedure) and a statistical gradient [61], and a random search method with self-learning [59]. The algorithm for solving the problem of searching for optimal parameters consists of the following steps.

1. The Monte Carlo procedure is performed: in the field of feasible solutions, $\overline{N}$ vectors are D_s-randomly selected that obey the normal distribution law. Moreover, for each of the vectors $\overline{N}$, the mathematical expectation is the original vector $\overline{x}^0$, and the dispersion is the set value.

2. The function $\overline{H}(x) = F(\overline{x}) R(\overline{x})$ for each vector $\overline{x}$ is calculated, where $F\left(\overline{x}\right)$ – the value of the objective function for the vector of variables $\overline{x}$ in this test:

$$R(\overline{x}) = \begin{cases} 1, \text{ если } \overline{x} \in D_s, \\ 0, \text{ если } \overline{x} \notin D_s. \end{cases}$$

3. The gradient of the function $H(\overline{x})$ is determined based on statistical data and the gradient refinement of the solution is obtained, which is obtained in statistical tests.

A process consisting of a series of tests is repeated many times. In this case, the Monte Carlo procedure ensures that the global extremum falls into the region, and the refinement of the solution by the gradient method accelerates the convergence process. After a given number of statistical tests, the range of the random vector of choice is substantially narrowed, and the solution is refined in the vicinity of the vector determining the global maximum.

The method of random search with self-training consists of optimizing the criterion (the mathematical expectation of a non-linear function $M\{F(\bar{x},\omega)\}$, which depends on the *n*-dimensional vector of variables $\bar{x}$ and the *l*-dimensional vector of random factors ω) through a series of local descents. The global search is provided by the forced departure of the minimum found from the zone of attraction.

The starting point x^0 of a local search in the region of permissive values is determined by the Monte Carlo method.

N_2-random steps $\bar{x}_p = \bar{x}^\circ + A\Sigma$ are taken from the test point x^0, where A is the diagonal matrix with the elements of the diagonal a_{ii}; $a_{i\varepsilon}$ – step module in the *i*-coordinate; Σ – a random vector whose coordinates are random numbers with a uniform distribution density on the interval [-1; 1]. From these samples, the best one that gives the function $M\{F(\bar{x},\omega)\}$ the largest increment in the direction of decrease (search for the global minimum) is selected. The current point of the search moves in the direction of the best sample with a step proportional *A*.

The efficiency of further searches is increased because the distribution density of the samples $P(\bar{x})$ during the search process is constantly changing so that it sequentially shrinks to the area suspected of an extremum. This is the learning process.

Specific examples of the implementation of the stochastic optimization problem, sets of output parameters $x = \{V, f, \gamma, \alpha ...\}$ for various values of the confidence probability of constraint fulfillment are given in [59; 60; 64].

4. MULTI-CRITERIAL OPTIMIZATION

4.1. Formulation of the problem multi-criteria optimization

In the case when the utility (effectiveness) of alternative solutions is evaluated by a known scalar function, methodological problems do not arise; only computational difficulties are possible. The situation is different with multi-criterion optimization problems, where the main question is the problem – what should be considered the best alternative in a problem with several objective functions that are contradictory and reach a maximum at various points of the set of permissive alternatives.

There are various ways to solve this problem. Let the choice of a solution be determined by the condition of maximum functionals $f_1(\bar{x}), f_2(\bar{x}), ..., f_n(\bar{x})$. In other words, it is necessary to find a solution that would provide maximum value to all optimization criteria, each of which characterizes a certain local quality of the alternative $\bar{x}$, for example, cost, reliability, performance, mass, etc.

The easiest way to solve this issue is to choose the main criterion (key factor) and transfer the rest to the category of criteria constraints. Difficulties are associated with the determination of reasonable boundaries for these constraints.

In several practical cases, a model can be used in which, instead of *n* various criteria, it is proposed to consider one criterion of the form

$$F(\bar{x}) = \sum_{i=1}^{n} c_i f_i(x); \quad \sum_{i=1}^{n} c_i = 1; \quad 0 \le c_i \le 1,$$

where c_i – positive numbers reflecting the degree of importance of the criteria; $f_i(\overline{x})$ – formed optimality criteria. Coefficients C_i – the result of an expert assessment of the decision-maker (DM). They reflect a compromise that satisfies the decision-maker.

Other models use a convolution of criteria in the form:

$$F(\overline{x}) = \prod_{i=1}^{n} f_i(x),$$

which is called the principle of fair compromise.

There are several methods for solving multicriteria problems. Note that the choice of method for solving a specific problem should be determined by the structure of a specific problem. Using analogies can lead to erroneous results.

The analysis of multicriteria tasks can be approached from other positions: you can try to reduce the set of initial options, i.e. excluded from the informal analysis of the deliberately worse decision options. Consider one of these paths proposed by the famous Italian economist Pareto in 1904 [50].

Suppose we made some choice. We denote it $\overline{x}$ and further suppose that there is another choice $\hat{\overline{x}}$, such that for all criteria $f_i(\overline{x})$ the inequalities

$$f_i\left(\hat{\overline{x}}\right) \geq f_i\left(\overline{x}^*\right).$$

The choice $\hat{x}$ is preferable $\overline{x}^*$. Therefore, all vectors satisfying $\overline{x}^*$ this inequality should be immediately excluded from consideration. It makes sense to engage in comparisons, to informally analyze only those

vectors $\overline{x}$ for which it does not exist $\hat{\overline{x}}$, for all criteria that satisfy the indicated inequalities. The set of all such values is called the Pareto set.

In the theory of decision making, there is the term "Pareto principle", which consists in the fact that as a solution to a multicriteria problem, one should choose only the vector that belongs to the Pareto set.

The Pareto principle does not separate a single solution, but only constrict the many alternatives. The final choice rests with the decision-maker. But the researcher, having built the Pareto set, facilitates the selection procedure. An example of such an analysis, where the search area for the optimal cost price solution (cutting modes) is significantly constricted, is considered in sub-section. 1.2. This allows you to effectively use the methods of analytical and statistical modeling in the presence of a large number of constraints in the problem of finding optimal cutting modes.

4.2. Optimization cutting modes using linear criteria convolution

Let two criteria for the effectiveness of the workpiece surface treatment process be given: a variable part of the cost price C and a variable part of the piece-accounting time t_{pa}. We represent the objective function as a linear convolution of these criteria:

$$\Phi = \beta_1 C + \beta_2 t_{pa};\ \ \beta_1 + \beta_2 = 1;\ \ \beta_1 \geq 0;\ \ \beta_2 \geq 0;$$

$$C = \frac{a + bV^m f^n d^k}{Vf};\ \ t_{pa} = \frac{1 + cV^m f^n d^k}{Vf};\ \ c = \frac{t_{ch}}{k_t C_t},$$

where β_1, β_2 – weights of the performance criteria importance that are appointed by the expert or group of experts or the decision-maker.

Let a constraint on the cutting force be given in the form of inequality:

$$k_{pz}C_{pz}V^{\alpha_1}f^{\alpha_2}d^{\alpha_3} \le P_{z\,pm}.$$

We assume that this restriction is active, i.e. takes the form of equality. To solve the problem of finding the optimal cutting modes, we use the method of Lagrange multipliers. We compose the Lagrange function:

$$\mathrm{L} = \Phi + \lambda\left(k_{pz}C_{pz}V^{\alpha_1}f^{\alpha_2}d^{\alpha_3} - P_{z\,pm}\right) \to \min,$$

where λ – Lagrange multiplier. Then the necessary minimum modes have the form [31]:

$$\frac{\partial \mathrm{L}}{\partial V} = \frac{bV^m f^n d^k (m-1) - a}{V^2 f}\beta_1 + \frac{cV^m f^n d^k (m-1) - 1}{V^2 f}\beta_2 + \lambda k_{pz}C_{pz}\alpha_1 V^{\alpha_1 - 1} f^{\alpha_2} d^{\alpha_3} = 0;$$

$$\frac{\partial \mathrm{L}}{\partial f} = \frac{bV^m f^n d^k (n-1) - a}{Vf^2}\beta_1 + \frac{cV^m f^n d^k (n-1) - 1}{Vf^2}\beta_2 + \lambda k_{pz}C_{pz}\alpha_2 V^{\alpha_1} f^{\alpha_2 - 1} d^{\alpha_3} = 0;$$

$$\frac{\partial \mathrm{L}}{\partial \lambda} = k_{pz}C_{pz}V^{\alpha_1}f^{\alpha_2}d^{\alpha_3} - P_{z\,pm} = 0.$$

Solving the reduced system of equations, we obtain:

$$\alpha_2\left[\left(bV^m f^n d^k (m-1) - a\right)\beta_1 + cV^m f^n d^k (m-1) - 1)\beta_2\right] =$$

$$= \alpha_1\left[\left(bV^m f^n d^k (n-1) - a\right)\beta_1 + \left(cV^m f^n d^k (n-1) - 1\right)\beta_2\right];$$

$$\alpha_2 V^m f^n d^k (m-1)(b\beta_1 + c\beta_2) - \alpha_2 (a\beta_1 + \beta_2) =$$

$$= \alpha_2 bV^m f^n d^k (n-1)(b\beta_1 + c\beta_2) - \alpha_1 (\alpha\beta_1 + \beta_2);$$

$$V^m f^n d^k = \frac{(\alpha_2 - \alpha_1)(a\beta_1 + \beta_2)}{(b\beta_1 + c\beta_2)\left(\alpha_2 (m-1) - \alpha_1 (n-1)\right)} = q.$$

It follows from the constraint that:

$$f = \left(P_{z0}V^{-\alpha_1}d^{-\alpha_3}\right)^{\frac{1}{\alpha_2}}; \quad P_{z0} = \frac{P_{z\,pm}}{k_{pz}C_{pz}}.$$

As a result, we get

$$V^m V^{-\frac{n\alpha_1}{\alpha_2}} d^{k-\frac{\alpha_3 n}{\alpha_2}} P_{z0}^{\frac{n}{\alpha_2}} = q;$$

$$V_0 = \left(qd^{k-\frac{\alpha_3 n}{\alpha_2}} P_{z0}^{-\frac{n}{\alpha_2}}\right)^{\frac{\alpha_2}{m\alpha_2 - n\alpha_1}};$$

$$f_0 = \left(qd^{k-\frac{\alpha_3 n}{\alpha_2}} P_{z0}^{-\frac{n}{\alpha_2}}\right)^{\frac{\alpha_1}{n\alpha_1 - m\alpha_2}} P_{z0}^{\frac{1}{\alpha}} d^{-\frac{\alpha_3}{\alpha_2}}.$$

Thus, for a given cutting depth t = const for specific values of $P_{z\,pm}$, β_1, β_2 it is possible to calculate the values of cutting speeds and feeds that provide the minimum of a generalized objective function Φ, which is a linear convolution of two performance criteria.

Consider an example. Let the model parameters for given cutting conditions:
a = 5 cent/min; t_{ch} = 2 min; a' = 15 cent/period; k_t = 1; $C_t = 4 \cdot 10^{12}$; m = 5; n = 2.25; k = 1; k_{Pz} = 1; C_{Pz} = 300; α_1 = -0.15; α_2 = 0.75; α_3 = 1; d = 3 mm; $P_{z\,pm}$ = 3000 N.

Assign specific values to the importance scales: β_1 = 0.6; β_2 = 0.4. Then the optimal values of the cutting speed and feed: V_0 = 115.7 m/min; f_0 = 0.09 mm/rev.

4.3. Optimization of cutting modes using the multiplicative convolution of criteria

Let it be necessary to optimize cutting conditions according to two performance criteria: productivity $P_r = \frac{1}{t}$ and processing cost price C.

The first should be maximized, and the second should be minimized. Let us convolute these two criteria into one efficiency criterion E_f, equal to the ratio of the maximized criterion to the minimized criterion, i.e., the objective function has the form:

$$E_f = \frac{P_r}{C} = \frac{\dfrac{\frac{1}{1+CV^m f^n d^k}}{Vf}}{\dfrac{a+bV^m f^n d^k}{VS}} = \frac{V^2 f^2}{(a+bV^m f^n d^k)(1+CV^m f^n d^k)}.$$

We believe that the constraints are set – equality on the main component of the cutting force P_z:

$$P_z = k_{pz} C_{pz} V^{\alpha_1} f^{\alpha_2} d^{\alpha_3} = P_{z\,pm}.$$

We determine the maximum of the function by the method of Lagrange multipliers:

$$\mathrm{L} = \frac{V^2 f^2}{a+(b+ac)V^m f^n d^k + bcV^{2m} f^{2n} d^{2k}} + \lambda(P_{z\,pm} - k_{pz} C_{pz} V^{\alpha_1} f^{\alpha_2} d^{\alpha_3});$$

$$a_1 = (a+ac)d^k; \quad a_2 = bcd^k;$$

$$\frac{\partial \mathrm{L}}{\partial V} = \frac{\left(2a_2 V^{2m} f^{2n}(1-m) + a_1 V^m f^n (2-m) + 2a\right)Vf^2}{\left(a + a_1 V^m f^n + a_2 V^{2m} f^{2n}\right)} -$$

$$-\lambda \alpha_1 k_{pz} C_{pz} V^{\alpha_1 - 1} f^{\alpha_2} d^{\alpha_3} = 0;$$

$$\frac{\partial \mathrm{L}}{\partial f} = \frac{\left(2a_2 V^{2m} f^{2n}(1-n) + a_1 V^m f^n (2-n) + 2a\right)V^2 f}{\left(a + a_1 V^m f^n + a_2 V^{2m} f^{2n}\right)^2} -$$

$$-\lambda \alpha_2 k_{pz} C_{pz} V^{\alpha_1} f^{\alpha_2 - 1} d^{\alpha_3} = 0;$$

$$\frac{\partial \mathrm{L}}{\partial \lambda} = P_{z\,pm} - k_{pz} C_{pz} V^{\alpha_1} f^{\alpha_2} d^{\alpha_3} = 0.$$

We multiply the first equation by $V\alpha_2$, the second by $f\alpha_1$ and subtract the second equation from the first. We get:

$$\alpha_2\left((1-m)2a_2 V^{2m} f^{2n} + (2-m)a_1 V^m f^n\right) + 2a\alpha_2 -$$

$$-\alpha_1\left((1-n)2a_2 V^{2m} f^{2n} + (2-n)a_1 V^m f^n\right) +$$

$$+2a\alpha_1 = 0;$$

$$2a_2' V^{2m} f^{2n} + a_1' V^m f^n = \frac{2a(\alpha_1 - \alpha_2)}{\alpha_2(1-m) - \alpha_1(1-n)} = h.$$

Denote $V^m f^n$ by x. Then:

$$2a_2' x^2 + a_1' x - h = 0 \rightarrow x = \frac{-a_1' + \sqrt{a_1' + 8a_2' h}}{4a_2'}.$$

Therefore, the optimal cutting speed equal $V_0 = f^{-\frac{n}{m}} x^{\frac{1}{m}}$. From the third equation we get the optimal feed:

$$P_{z\,pm} = k_{pz} C_{pz} f^{-\alpha_1 \frac{n}{m}} x^{\frac{\alpha_1}{m}} f^{\alpha_2} d^{\alpha_3}; \quad f_0 = \left(P_{z0} x^{-\frac{\alpha_1}{m}} d^{-\alpha_3} \right)^{\frac{m}{\alpha_2 m - \alpha_1 n}}.$$

We calculate the values of the objective function and optimal cutting conditions for the conditions given in the previous subsection. Get: h = 2.28; x = 2.32·10^{12}; f_0 = 0.72 mm/rev; V_0 = 349.5 m/min; Φ = 259.1 units.

4.4. Using the method of sequential climb-down when optimizing cutting conditions

To find the optimal modes of finishing turning the shaft (the machining scheme is shown in Fig. 2.1), ensuring a minimum machine time and a minimum detail bending.

Initial data: workpiece – shaft; machining section – 1; operation – finishing turning; machine tool - screw-cutting lathe 16K20F3; the type and size of the workpiece – rolled C45 (steel 45, σ_B = 598 MPa), D = 100 mm, R_a = 2.5 μm (R_z = 10), L_d = 860 mm, L_1 = 250 mm, allowance Δ = 1 mm. The tool is a straight turning cutter with mechanical fastening of a hexagonal tip made of DIN HS 123 (T15K6 hard alloy); φ = 45^0; φ_1 = 10^0; wedge transition radius r = 1.0 mm; γ = 10^0; T = 60 min; d = Δ = 1 mm.

Following the algorithm of the climb-down method, it is necessary to implement several stages [66]:

1. Rank the particular performance criteria by introducing the priority of the productivity criterion, expressed in terms of machine time. $t_m = \Phi_1$

2. Find the minimum value of the machine time of shaft machining under constraints on the cutting capabilities of the tool $V \le V_T$ and the acceptable value of surface roughness $R_z \le R_{z\,pm}$.

When formulating the problem and creating constraints, we use the constraint formation technique [8] and the geometric programming method of zero degrees of difficulty [15]. We consider the GP problem in a direct formulation.

Minimize: $\Phi_1 = t_m = 250 \cdot f^{-1} n^{-1}$,

under restrictions: $0.0017 \cdot f^{0,2} n \le 1$;

$2.04 \cdot f^{0,7} n^0 \le 1$.

It should be noted that in this problem there are three posynomial terms and two variables $\{f;\, n\}$, which characterizes it as a problem of a zero degree of difficulty (the number of posynomial terms is one more than the number of variables). Also, each constraint contains a single posynomial term. This direct formulation of the GP problem corresponds to the dual GP problem with constraint [15].

Dual setting:

Maximize: $V(W) = 250^{w_{01}} \cdot 0.0017^{w_{11}} \cdot 2.04^{w_{21}}$,

under normalization constraints: $w_{01} = 1$

and orthogonality: $f: -w_{01} + 0.2 w_{11} + 0.7 w_{21} = 0;$ (4.1)

$n: -w_{01} + w_{11} = 0.$ (4.2)

Using the appropriate GP program of zero degrees of difficulty, we determine:

- values of dual variables: $w_{01} = 1;\ \ w_{11} = 1;\ \ w_{21} = 1.14;$
- dual function maximum: $\max V(w) = 250 \cdot 0.0017 \cdot 2.04^{1.14} = 0.96.$

Optimal values of cutting modes elements from the conditions of invariance [10]:

$$\begin{cases} 0.96 = 250 \cdot f^{-1} n^{-1}; \\ \dfrac{1}{1} = 0.0017 \cdot f^{0,2} n; \\ \dfrac{1.14}{1.14} = 2.04 \cdot f^{0,7} n^{0}. \end{cases}$$

The only solution to this system of equations is:
f_1^0 = 0.36 mm/rev; n_1^0 = 717.37 rpm; V_1^0 = 225.25 m/min.

It must be remembered that the maximum of the dual function according to the GP theory corresponds to the minimum of the initial objective function, i.e. $t_m = 0.96$ min.

3. Solve the optimization problem by the second criterion of effectiveness Φ_2, i.e. find such cutting modes $\{f; n\}$, that would minimize the detail bending arrow under the same constraints on the cutting capabilities of the tool and the permissive roughness (see step 2).

Consider the statement of the problem by the GP technique and [15]:

$$\Phi_2 = f_2 = 0.01 n^{-0.3} f^{0.6} \to \min; \qquad (4.3)$$
$$0.0017 \cdot f^{0,2} n \le 1;$$
$$2.04 \cdot f^{0,7} n^{0} \le 1.$$

In this setting, the GP method is unacceptable, since there is no significant change for the line *f* of dual variables [67], which leads to the

appearance of negative dual weight w_{21}. This contradicts the initial constraint of the GP method: $w \ge 0$. Analysis of various machining schemes showed that this situation is typical for various combinations of initial data and various machining schemes (turning with transverse feed, threading, etc.), which are reduced to the setting (4.3).

To solve the problem of finding the minimum bending arrow f_b, we use the linear programming method [3]. A mathematical model after reduction to a linear form is a set of linear inequalities and an objective function f_2 obtained as a result of the logarithm of the expression for the initial constraints and constraints on the kinematic capabilities of the machine 16K20F3:

$$(f_{\min} \le f \le f_{\max};\quad n_{\min} \le n \le_{\max});$$
$$x_1 + 0.2x_2 \le 6.38;$$
$$0.7x_2 \le -0.71;$$
$$x_1 \le 7.6;$$
$$x_1 \ge 2.53;$$
$$x_2 \le 1.03;$$
$$x_2 \ge -3.0;$$
$$f_2 : -4.61 - 0.3x_1 + 0.6x_2 \to \min. \qquad (4.4)$$

The notation $\ln n = x_1,\ \ln f = x_2$ is introduced here. To implement the graphical method, as the most obvious, the above inequalities and equations should be represented by straight lines in the coordinate system *n*–*f* (Fig. 4.1). On the graph, you need to draw lines in double logarithmic scales and indicate with arrows which side of each line points are corresponding to the permissible values of *n* and *f*.

In this problem, the constraint system is compatible, the admissible region ABCD (Fig. 4.1) is a closed polyhedron, and dependence (4.4) to be minimized is shown by a dashed line. This linear function will be minimal when it passes through point *B* of the possible solutions polygon. The coordinates of this point will give the optimal solution: f^0 = 0.05 mm/rev; n^0 = 1980 rpm; bending arrow f_b^0 = 0.00017 mm; value of machine time t_m = 2.53 min.

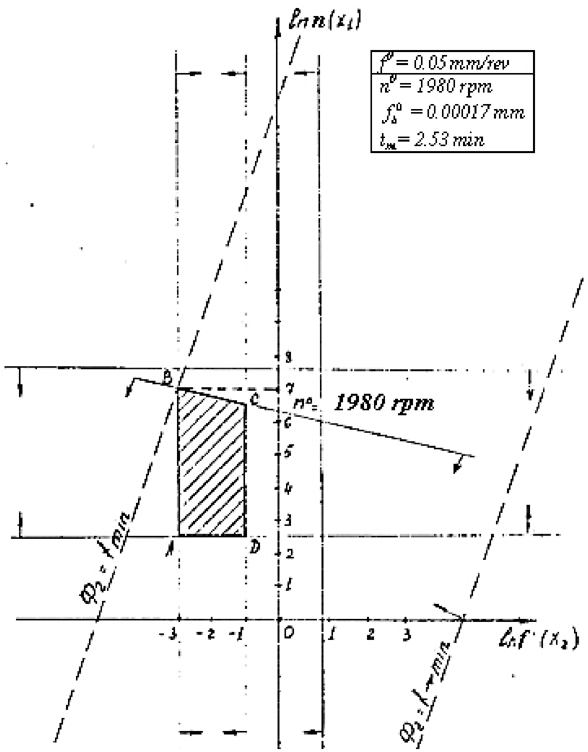

Fig. 4.1. The scheme of finding the optimal cutting conditions by criterion f_b

An analysis of the results shows that the desire to minimize the machine time and the detail bending arrow leads to two sets of optimized parameters and two different values of the machine time:

$$\left\{f_1^0 = 0.36;\ n_1^0 = 717.37\right\};\quad \left\{f_2^0 = 0.05;\ n_2^0 = 1980.0\right\}.$$

To find a compromise option for the implementation of machining modes, it is necessary to perform the fourth stage of solving the problem by the climb-down method.

4. Choosing a climb-down according to the first criterion of efficiency t_m. When choosing a climb-down, you should compare the value of machine time obtained as a result of single-criterion optimization (step 2) with the “tabular” value of machine time. For this processing case, by the data [37], we obtain tabular values:

$$f^T = 0.25 \text{ mm/rev};\ V^T = 268 \text{ m/min};\ t_m = 1.17 \text{ min}.$$

Then the climb-down according to the first criterion is defined as the difference between the table and optimal (according to the first criterion) machine time:

$$h_1 = t_м^T - t_{м1} = 1.17 - 0.96 = 0.21 \text{min}.$$

5. Find the minimum value of the detail bending arrow $f_2 = \Phi_2$, but with a new system of constraints, including the initial constraints on the cutting capabilities and the allowable roughness of the machined surface, as well as an additional constraint imposed on the value of the first criterion: $\Phi_1 \le \Phi_1^{**}$, where $\Phi_1^{**} = \Phi_1 + h_1 = 0.96 + 0.21 = 1.17$ min. This criterion transfer technique in constraint in another context has been considered. You should compare two options for the same technique and identify the difference in their use.

For this option, the search for an extremum f_2 is possible using the GP method of the first degree of difficulty [15]. Consider the mathematical model of the problem at this stage.

Direct statement of the problem:

Minimize: $\Phi_2 = f_2 = 0.01 \cdot n^{-0.3} f^{0.6} \to \min$;

under constraints:

$$0.0017 \cdot f^{0,2} n \le 1;$$

$$2.04 \cdot f^{0,7} n^0 \le 1;$$

$$213.68 \cdot f^{-1} n^{-1} \le 1.$$

Dual statement of the problem:

Maximize: $V(w) = 0.01^{w_{01}} \cdot 0.0017^{w_{11}} \cdot 2.04^{w_{21}} \cdot 213.68^{w_{31}}$;

under constraints: $w_{01} = 1$;

$$-0.3w_{01} + w_{11} - w_{31} = 0;$$

$$0.6w_{01} + 0.2w_{11} + 0.7w_{21} - w_{31} = 0. \quad (4.5)$$

To solve the GP problem of the first degree of difficulty, it is recommended to use a machine-oriented dual method [34]. To do this, we choose the basic dual weights $w_i = \{w_{01}, w_{11}, w_{21}\}$ and solve the system of equations (4.5) to the basis variables [the number of basis weights is one more than the number of optimized parameters and is equal to the number of system equations (4.5)]. In the case under consideration, linear dependences on w_{31}:

$$w_{11} = w_{31} + 0.3; \quad w_{21} = 1.14w_{31} - 0,94.$$

Wherein $w_{01} = 1$ (according to the normalization condition).

For everyone w_i to be positive, there w_{31} must be more than 0.82. As a first approximation, we take w_{31} = 0.82. We calculate the maximum value of the dual function: $V(w_{31})$:

$$V(w_{31}) = 0.01 \cdot 0.0017^{1.14} \cdot 2.04^0 \cdot 213.68^{0.82}.$$

The maximum of the dual function is equal to the minimum of the "direct", i.e. $f_2 = 0.000656$, which is significantly lower than the permissible detail climb-down ($f_2 = 20\%$ tolerance fields and at $R_z = 10 \to f_{2\,\text{pm}} = 0.02$ mm).

To search $\max V(w_{31})$, you can use the program of numerical optimization (dichotomy, golden ratio, Fibonacci). The appeal to the standard dichotomy program is described in [68-70]. To implement the machine search procedure, it is necessary to express $V(w)$ as a function of w_{31} and use the optimum search program. For this statement (4.5) and the values $w_{31} = 0.82$: $V(w_{31}) = 0.000769 \cdot 0.82^{w_{31}}$. For this function, the largest upper bound corresponds to the minimum value $w_{31}(w_{31} = 0.82)$; $V(w_{31}) = 0.000656$.

From the conditions of partial invariance for dominant weights w_{01} and w_{11}:

$$\begin{cases} 0.000656 = 0.01 \cdot n^{-0.3} f^{0.2}; \\ \dfrac{1}{1} = 0.0017 \cdot f^{0,2} n, \end{cases}$$

we get the values of the optimized parameters: $f_3^0 = 0.28$ mm/rev; $n_3^0 = 756.56$ *rpm*; $V_3^0 = 237.56$ m / min and the value of machine time

$t_m = 1.17$ min. Thus, as a result of applying the climb-down method, a compromise solution has been found: $f_3^0 = 0.28$; $n_3^0 = 756.56$, which provides the greatest approximation of both criteria t_m and f_2.

4.5. Using the "Ideal Point" Method when optimizing cutting conditions

Find the optimal modes of the shaft rough turning (the diagram is shown in Fig. 2.1), providing a minimum of machine time t_m, the minimum detail bending f_2 and the minimum cutting capacity N [71].

Initial data: workpiece – shaft; machining section – 2; machine – screw-cutting lathe 16K20F3; type and size of the workpiece – rolled, 5130 (steel 30X, $\sigma_B = 883$ MPa), D = 95 mm; $R_z = 40\,\mu$m; $L_d = 860$ mm; allowance $\Delta = 2$ mm. Tool – a turning lathe straight assembly tool with the mechanical fastening of a hexagonal plate made of DIN HS021 (T30K4 hard alloy), T = 60 min.

Following the algorithm of the ideal point method, the following steps should be implemented [21].

1. Calculate the extreme (in this statement, the minimum) values of the criteria $\Phi_i(f, n)$, i.e. solve three extreme problems (where i = 3 – number of particular performance criteria).

Extreme task 1. To minimize machine roughing time $\Phi_1 = t_m$ with constraints on the cutting capabilities of the tool $V \le V_T$ and the capacity of the electric motor drive the main movement of the machine ($N_c \le N_e$). In the last constraint: N_c – power required for cutting, kW:

$$N_c = \frac{P_z V}{60 \cdot 1020} = \frac{C_{pz} K_{pz} f^y d^x (\pi D n)^{n_p+1}}{1000^{n_p+1} 60 \cdot 1020},$$

where N_e – effective capacity of the machine, kW; $N_e = N_m \eta$; N_m – drive power of the main movement, given in the passport of the machine; η – efficiency of transmission from the electric motor to the tool. We use the geometric programming method for searching $\min t_м$ and optimal cutting conditions (n^0, f^0).

Direct statement of the problem:

Minimize: $t_m = 250 \cdot f^{-1} n^{-1}$;

under constraints: $0.0019 \cdot n f^{0.45} \le 1$;

$$0.06 \cdot n^{0.85} f^{0.9} \le 1.$$

For this problem of zero degrees of difficulty, we form a dual statement:

Maximize: $V(w) = 250^{w_0} \cdot 0.0019^{w_1} \cdot 0.006^{w_2}$;

under constraints: $w_0 = 1$;

$$n: -w_0 + w_1 + 0.85 w_2 = 0;$$

$$f: -w_0 + 0.45 w_1 + 0.9 w_2 = 0.$$

As a result of solving the GP problem in a dual statement, we obtain the following values of the dual variables: $w_0 = 1$; $w_1 = 0.1$; $w_2 = 1.06$ and the extremum of the objective function (the maximum of the dual function, which according to the rules of geometric programming is equal to the minimum of the direct function t_m)

$$V(w) = 250 \cdot 0.0019^{0.1} \cdot 0.006^{1.06} = 0.583 \text{ min}.$$

Based on the invariance conditions (for one-term posynomials), we determine the optimal cutting modes:

$$1 \cdot 0.583 = 250 \cdot f^{-1} n^{-1}; \;\; 1 = 0.0019 \cdot f^{0.45} n; \;\; 1 = 0.006 \cdot f^{0.9} n^{0.85}.$$

The only solution to this system of equations is:

$$f_1^0 = 0.7 \text{ mm/rev}; \;\; n^0 = 603.8 \text{ rpm}; \;\; V_1^0 = 180.2 \text{ m/min}.$$

Extreme task 2. Minimize the details bending arrow: $\Phi_2 = f_2$:

$$f_2 = \frac{P_y l^3}{70 E\Im} = \frac{1.98 \cdot (0.1\pi)^{-0.3} \cdot 860^3 \cdot 2^{0.9} n^{-0.3} f^{0.6}}{70 \cdot 20000 \cdot 0.05 \cdot 100^4} = 0.0484 \cdot n^{-0.3} f^{-0.6} \to \min,$$

with the same constraints on:

cutting capabilities: $0.0019 \cdot n f^{0.45} \le 1$;

cutting capacity: $0.006 \cdot n^{0.85} f^{0.9} \le 1$.

It should be verified once again that, in such a formulation, the optimization problem due to the appearance of negative dual weights cannot be solved by the method of geometric programming. To solve this problem, we use the linear programming method. The linear model of extremal problem 2, taking into account the constraints on the kinematic capabilities of the machine 16K20F3 $(12.5 \le n \le 2000;\; 0.05 \le f \le 2.5)$, represents the following system of inequalities:

$$x_1 + 0.45 x_2 \le 6.24; \qquad (4.6)$$

$$0.85 x_1 + 0.9 x_2 \le 5.12; \qquad (4.7)$$

$$x_1 \le 7.6; \qquad (4.8)$$

$$x_1 \ge 2.53; \qquad (4.9)$$

$$x_2 \le 1.03; \qquad (4.10)$$

$$x_2 \ge -3.0, \qquad (4.11)$$

and objective function: $\Phi_2 = -3.03 - 0.3 x_1 + 0.5 x_2 \to \min$. (4.12)

The notation is introduced here: $ln\, n = x_1$; $ln\, f = x_2$.

In this problem, the constraint system is compatible, the admissible region ABCED is a convex closed polygon, and the dependence (4.12) to be minimized is shown by a dashed line Φ_2 (Fig. 4.2). This linear function is minimal when passing Φ_2 through point B of the possible solutions polygon. The coordinates of this point give optimal values: $n_2^0 = 1980.92$ rpm; $f_2^0 = 0.05$ mm/rev.

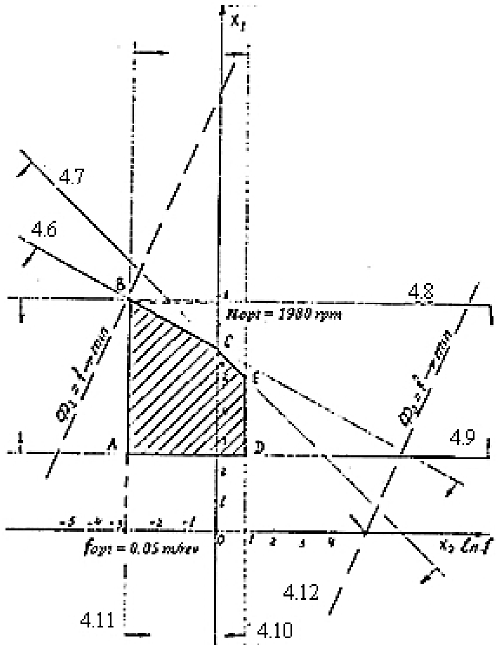

Fig. 4.2. The scheme of finding the optimal cutting modes by criterion f_2

Extreme task 3. Minimize cutting capacity $\Phi_3 = N_c$:

$$N_c = \frac{P_z V}{1020 \cdot 60} = \frac{10 \cdot 300 \cdot 0.98 \cdot 2 \cdot (0.1\pi)^{0.85} n^{0.85} f^{0.75}}{1020 \cdot 60} = 0.03 \cdot f^{0.75} n^{0.85} \to \min$$

subject to constraints on the cutting and kinematic capabilities of the machine 16K20F3. Since geometric programming in this statement is not applicable (see extremal problem 2), we use the linear programming method to search for optimal cutting conditions by criterion Φ_3.

Linear task model:

$$x_1 + 0.45 \cdot x_2 \le 6.24;$$
$$x_1 \le 7.5;$$
$$x_1 \ge 2.53;$$
$$x_2 \le 1.03;$$
$$x_2 \ge -0.3;$$
$$\Phi_3 = N_c = -3.51 + 0.85 x_1 + 0.75 x_2 \to \min. \quad (4.13)$$

The polygon of feasible solutions for this problem is formed by lines (4.6), (4.8), (4.11), and the linear function (4.13) will be minimal when the dashed line passes through the point A' (Fig. 4.3). The coordinates of this point are the optimal values: $n_3^0 = 12.5$ rpm; $f_3^0 = 0.05$ mm/rev.

2. Form normalized differences $\Delta_i(x) = \Delta_i(n, f)$.

By the algorithm of the ideal point method, all particular criteria should be transformed into such a way that the requirements for them are expressed in the desire to increase them. If any criterion is desired to be reduced, then formally one can introduce the criterion $1/\Phi_i(x)$, that needs to be increased. With maximized criteria $\Delta_i(n, f)$, a reduced difference is formed:

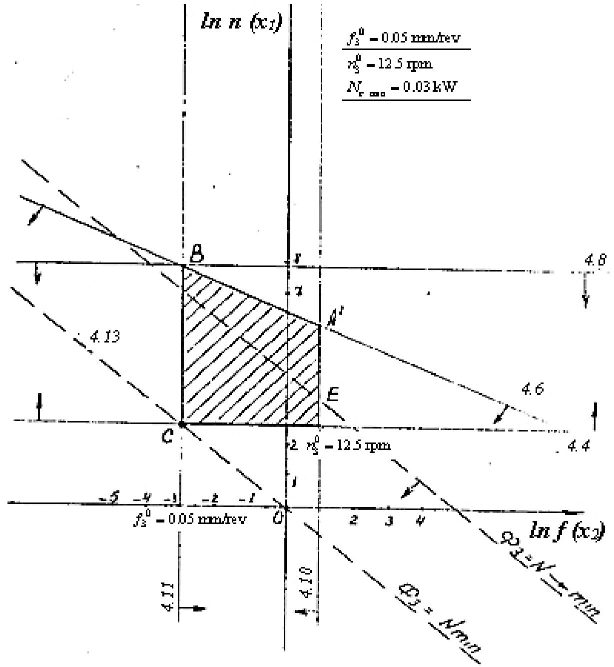

Fig. 4.3. The scheme of finding the optimal cutting modes by criterion N_c

$$\Delta_i(n, f) = \left[\Phi_i^* - \Phi_i(n, f)\right] / \Phi_i^*.$$

Each such difference shows how the object with the proposed technological parameters (n, f) differs from the ideal variant by the i_{th} criterion [72]. Differences $\Delta_i(n, f)$ form a new system of particular criteria equivalent to the original $\{\Phi_i\}$ system, each of which is desirable to reduce.

For this three-criteria problem, when constructing such normalized differences, it is necessary to calculate the values of particular criteria at any three arbitrary (in the general case) points in the region of admissible values. We accept as required: point A (ideal point by criterion Φ_1), point B (ideal point by criterion Φ_2). Values $\Delta_i(n, f)$ in points *A, B, C* are given in the table. 4.1.

Table 1

Normalized differences in optimal cutting modes

x_i	Φ_i		
	Φ_1	Φ_2	Φ_3
A	0	0.89	0.995
B	0.77	0	0.986
C	0.998	0.78	0

3. To form from the criteria $\Delta_i(x)$ a convolution $\omega(x, \lambda)$ of the type of generalized logical conjunction with weight coefficients λ_i: $\omega(x, \lambda) = \max \lambda_i \Delta_i(x)$. Weighted normalized differences $\lambda_i \Delta_i(x)$ in points *A, B, C* for the case of distribution of weights λ: $\lambda_1 = 0.4$; $\lambda_2 = 0.1$; $\lambda_3 = 0.5$ are given in Table. 4.2. For such a distribution: $\{\omega(A) = 0.498; \quad \omega(B) = 0.493; \quad \omega(C) = 0.4\}$.

Table 4.2

Weighted normalized differences

x_i	Φ_i		
	Φ_1	Φ_2	Φ_3
A	0	0.09	0.498
B	0.31	0	0.493
C	0.4	0.08	0

4. Construct a minimax criterion ω:

$$\omega = \min_{x \in X} \max_{1 \le i \le n} < 0.498;\ 0.493;\ 0.4 > \min; \quad \omega(A) = 0.4;$$

As a result of the four stages, a technological option $\omega(A)$ was obtained that is closest in quality to ideal points according to particular criteria Φ_1, Φ_2, Φ_3 (taking into account weights: $\lambda_1 = 0.4$; $\lambda_2 = 0.1$; $\lambda_3 = 0{,}5$). The optimal variant coincides in this case with the variant of optimization for Φ_1 and is characterized by the values:

$$f^0 = 0.7\, mm / rev; \quad n^0 = 603.8\ rpm; \quad V^0 = 180.2\ m / \min.$$

5. OPTIMIZATION OF THE CUTTING PROCESS IN AUTOMATED CONDITIONS PRODUCTION

5.1. Brief theoretical information

In modern machine-building production, where flexible automated means of storage, processing, transportation and loading play an increasingly important role, the time balance necessary to turn the workpiece into a finished detail is redistributed [73-75]. If for universal (non-automated) equipment the detail is directly processed only 5% of the total time (the rest of the time is stored in the warehouse, at the machine, it is controlled, transported, etc.), then under the conditions of the functioning of flexible production systems (FPS), the detail located on the machine up to 90% of the total time balance, i.e. productivity at the same time increases by 15-20 times.

According to the forecast of SIRP (International Institute of Engineering Technology), starting in 2020, at least 30% of the manufactured machines will be embedded in automated processing systems, and in the future this percentage will increase.

In connection with the increase in the cost of FPS equipment, the complexity of the manufactured products themselves, the urgent task of creating reliable adequate models of the cutting process on CNC machines, machining centers (MC), FPS and, based on these models, the implementation of a comprehensive optimization of cutting modes, parameters cutting tool and machining conditions.

The specificity of the created model of the cutting process is due to the characteristic features of software-controlled equipment. These include:

- the increased rigidity of CNC machines in comparison with universal ones;
- the best heat removal from the cutting zone;
- combining several stages of processing (for example, roughing and finishing turning) without reinstalling;
- increased requirements for chip breaking and chip removal from the cutting zone;
- presence of spindle angular position sensors, with the help of which the spindle rotation and feed movement are synchronized;
- a discrete cycle of using the same tool (with intermediate storage in the tool store);
- a prompt adjustment of cutting modes (especially with the "CNC-control" system) and, as a result, the quick changeover to a new product;
- processing simultaneously from 4-5 sides due to the rotation of the table of the machining center;
- simplification of sets of clamping accessories (for example, axial machining without conductive bushings);
- the presence of built-in adaptive control systems according to the principle of limit regulation with optimization.

The task of optimal control of the cutting process on software-controlled equipment can be divided into two stages:

- substantiation of values chosen for the parameters of the modes that satisfy the system of constraints and ensure the extremality of the criterion for the quality criterion (efficiency) of the process. This task is called external optimization;
- control of processing parameters to maintain the optimal value of the quality criterion under the influence of disturbing influences on the process – the task of internal optimization.

When forming optimization models of the cutting process on the CNC machine, MC, FPS, it is necessary to take into account several requirements for the type and values of both controlled parameters and quality criteria. This, first of all, concerns the index of tool life and elements of cutting modes.

In [76-78], the range of variation of the tool life period is indicated, within which any value T can be considered optimal (T_0). This range is limited by the highest (maximum) productivity, on the one hand (the corresponding tool life is indicated T_{pr}), and the minimum cost, on the other (T_c):

$$T_0 = T_{pr} + \alpha(T_c - T_{pr}); \qquad \alpha = (1...9.63)\cdot 10^{-6}(C_m - N_c),$$

where C_m – the cost of the machine; N_c – correction factor for the cost.

Moreover, the higher the value C_m, the closer it should be T_0 to T_{pr}. Economic tool life T_c decreases with the increasing cost of the machine, but at the same time remains constant. The values T_{pr} characteristic of various models of CNC machines [14] is given.

When assigning cutting modes to MC, ensuring a minimum cost price, the condition must be performed

$$\frac{T_{n1}}{T_{e1}} + \frac{T_{n2}}{T_{e2}} + ... + \frac{T_{ni}}{T_{ei}} = 1, \qquad (5.1)$$

where T_{ni} – conditional economic stability of the i-th tool in the conditions of single-tool processing, adopted according to the norms [30]; T_e – conditional economic stability of the i-th tool with multi-tool processing at the MC.

Failure to comply with conditions (5.1) increases the cost price of processing due to frequent tool changes when working with increased

cutting speeds or increases the main time t_c of the i-th pass when working with low cutting speeds.

Another requirement for the range of optimal values is associated with the simultaneous dulling of all tools adjustment, which allows for the change of the MC tool storage as a whole. This normalization is expressed as follows:

$$T_{e1} / t_{c1} = T_{e2} / t_{c2} = ... = T_{ei} / t_{ci} = Q,$$

where Q – conditional economic tool life price, pcs.

For CNC and MC machines, it is advisable in some cases to increase the cutting speed by reducing the tool life period. This increase is facilitated by off-machine grinding, fine-tuning and adjustment of the tool with high accuracy, the ability to automatically change the rotational speed according to the program, and increased rigidity of the machine system [79]. So, for MC, it is recommended to increase the cutting speed by 15-100%, which increases the value of the machining productivity criterion [80].

In the calculations of the optimal cutting conditions (V, f, d), economic (most often the technological cost price), technical and economic (accuracy, roughness, etc.) and energy criteria (specific energy intensity, etc.) are used.

There are some cases in fairly narrow ranges of control parameters when the choice of the optimality criterion does not matter, since it is determined by the boundary of the permissible values of the optimized parameters, i.e. completely dependent on technological constraints.

However, it should be noted that in the formation of economic and technical-economic parameters, difficulties arise in a formalized description of the cutting process. This primarily relates to tool life models (tool life is a component in the criteria of cost price, productivity, etc.). Besides, these criteria, on the one hand, take into account the factors of a particular production (type, condition of equipment used, production tooling, tool size, etc.), and on the other hand, they do not directly take into account the accuracy, roughness of the treated surface and properties detail surface layer. Optimization using energy criteria does not directly depend on the economy of production, but reveals reserves of productivity and processing quality, which are predetermined by the physical characteristics of the interaction of the tool and detail material. As energy criteria for physical optimization are used:

- the wear rate of the cutting tool;
- cutting temperature corresponding to a minimum of tool wear [3];
-specific energy intensity η_1 characterizing the total energy consumption of chip formation reduced to a unit volume of the removed layer;
- latent energy of the surface layer deformation of the detail used in the optimization of finishing machining;
- metal removal per unit of capacity.

The benefits of energy criteria include [59]:

a) the ability to reflect the physical and mechanical state of the cutting zone;

b) universality in the sense of suitability for solving problems in various production conditions;

c) the ability to compare technological processes that are different and, as a result, use them for structural optimization;

d) reducing the problem of choosing the optimal technological conditions of cutting only to calculations using a computer without labor-intensive experimental studies;

e) the ability to process with maximum efficiency. As you know, cutting with a wedge-shaped tool is extremely energy inefficient - the total energy consumption is 8-10 times or higher than the cost of the useful work of forming a new surface. Since excess energy creates an increased dynamic and thermal tension of the cutting process, the

processing conditions corresponding to the minimum energy intensity increase the dynamic stability of the cutting process (by increasing the feed, cutting speed and rake angle of the tool). As a result of this, the level of vibrations in the machine system decreases and the tool life increases;

- f) maintaining a stable monotonous character of changing values η_1 and η_2 their statistical characteristics under the action of random factors. This makes the energy criteria acceptable in the problems of statistical optimization of the cutting process, as well as in adaptive control systems with optimization. Under the same conditions, there is a “floating” dependence of cost price and productivity on cutting and feed rates and a change in their extreme values over a fairly wide range.

Features of processing on machines with program control are taken into account by the relevant constraints on the machining process.

1. Constraint on the tool life of the cutting tool.

When using energy criteria for optimizing the cutting process on CNC machines, the turning parameters should be set so that the cutter tool life period corresponds to the machine time of one pass or a multiple *K* number of them:

$$T \geq K \frac{\pi DL}{1000 \cdot Vf}. \tag{5.2}$$

Condition (5.2) does not contradict the target setting of minimizing the cost price *C* (it is achieved with fewer number passes) and maximizing productivity (the prerequisites for the rational operation of the cutting tool are created). Along with cost price reduction, the implementation of this constraint provides an increase in quality and a decrease in the laboriousness of detail manufacturing, since there is no need for an additional operation of removing the marks on the treated surface. As the lower bound of the permissible tool life, the tool life T_{pr}

can be taken at which the greatest productivity of the operation is achieved:

$$T \geq T_{pr}. \tag{5.3}$$

2. Constraint on the cutting force allowed by the rigidity of the CNC machine tool system.

Based on the analysis of the factory conditions for processing parts of various classes, [59] the cutting forces for CNC machines of the turning group were found, which is the maximum allowable from the rigidity condition of the machine system [*P*]:

$$P \leq [P]. \tag{5.4}$$

For most CNC lathes, the maximum allowable cutting force depends on the rigidity of the fastening of the cutter in the tool holder (cut-out section). When turning non-rigid details, value [*P*] can be specified from the conditions of precision machining. Values [*P*] and some other normative technological parameters are given in [30].

3. Constraint on the accuracy of processing.

The greatest influence on the accuracy of machining details has dimensional wear of the cutter:

$$\Delta \geq h_r / ctg\alpha, \tag{5.5}$$

where Δ – tolerance on the size, mm; h_r – the height of the wear area on the rear surface of the tool, which can be estimated by the allowable value [h_r], adopted according to the standards [48]; α – the main rear corner of the tool.

4. Constraint on the roughness of the treated surface R_a.

It must comply with the requirements of the technology and not exceed the value [R_a] specified in the detail drawing (operational sketch):

$$R_a \leq [R_a].$$

5. Parametric constraints on cutting conditions and the geometry of the cutting part of the tool (x_i):

$$\left\{\begin{matrix} n_{m.\min} \leq n \leq n_{m.\max}; \quad f_{m.\min} \leq f \leq f_{m.\max}; \quad \gamma_{\min} \leq \gamma \leq \gamma_{\max i} \\ \alpha_{\min} \leq \alpha \leq \alpha_{\max}; \quad \varphi_{\min} \leq \varphi \leq \varphi_{\max} \end{matrix}\right\},$$

where γ and φ – respectively, the main rake angle and the frontal approach angle of the cutting tool.

The set of formed constraints on the controlled parameters of the cutting process and the criterion (criteria) of efficiency make up the mathematical model of optimization.

Depending on the type of processing (rough, finishing), the number of passes, there may be several modifications of mathematical models with an energy quality criterion.

For preliminary passes, there is a tendency to remove the maximum possible allowance permitted by the rigidity of the machine system and the requirements for tool life. The conditional energy intensity of the process $\eta_1(x_i)$ is taken as a criterion. The mathematical model, in this case, is [59] a system:

$$\begin{cases} \Phi = \eta_1(x_i) = \dfrac{U}{Vfd} \to \min; \\ T \geq t_m; \\ T \geq T_{pr}; \\ \rho \leq [\rho]; \\ x_i^{\min} \leq x_i \leq x_i^{\max}. \end{cases} \tag{5.6}$$

If the entire stock is removed with one cutter during the tool life period, then another modification of the model is considered taking into account the accuracy constraints:

$$\begin{cases} \Phi = \eta_1(x_i) = \dfrac{U}{Vfd} \to \min; \\ T \ge \sum_{i=1}^{N_n} t_{mi}; \\ \rho_i \le [\rho]; \\ [\Delta] \le \Delta_i \le \Delta_{i-1}; \\ [R_a] \le R_{ai} \le R_{a(i-1)}; \\ x_i^{\min} \le x_i \le x_i^{\max}. \end{cases} \qquad (5.7)$$

For the final pass, the requirements of single-pass processing (5.7) are preserved with the addition of a constraint on the hardening depth *H* of the surface layer:

$$H(d_{N_n}) \ge H(d_{N_{n-1}}) + d_{N_n},$$

where d_{N_n} – is the cutting depth at the N_n – th pass.

5.2. Optimization implementation example optimal search procedures cutting modes on CNC machines

Find the optimal modes of the rough turning of the shaft (the processing scheme is shown in Fig. 2.1), providing a minimum of specific energy consumption η_1.

Initial data: lathe with CNC 16K20F3; stock – rolled, DIN 41 Cr4 (steel 40X, σ_B = 981 MPa, $\theta = 45^0$, D = 100 mm, R_z = 80 µm machining allowance Δ = 3 mm; tool – a cutter turning lathe with mechanical fastening of a hexagonal plate from a DIN HS123 (hard alloy T15K6),

cutter geometry: $\varphi = 45^0$, $\varphi_1 = 10^0$, $\gamma = 5^0$, transient radius of a wedge r = 1 mm. The tool life period is not set.

The formation of the objective function

As an optimization criterion, we take the specific energy intensity of the process η_1, the analytical description of which is [14]:

$$\eta_1 = \frac{U}{Vfd} = \frac{0.6\sigma_B}{(1-1.7\psi)\cos\gamma\cos\alpha}\left[\frac{\xi - \sin\gamma}{\cos\gamma} + tg\theta\right],$$

where ψ – the relative narrowing, %; ξ –chip thickness compression ratio; for steels the angle of inclination for reference shear plane $\theta = 45^0$. The chip thickness compression during turning of high-strength steels can be represented by a generalized empirical dependence [14]:

$$\xi = 1 + 81.22\sigma_B^{-0.849} V^{-0.113} f^{-0.372} d^{0.217} \sigma^{-0.0546}.$$

After the transformations, we obtain the following analytical expression for:

$$\eta_1 = 5118.26 + 398.42 \cdot V^{-0.113} f^{-0.217} \to \min. \qquad (5.8)$$

Formation of a system of technical limitations

Since during roughing the greatest influence on the choice of cutting modes is exerted by the force characteristics, then we will take the system of constraint (5.6) as the main one. We transform the inequality constraints according to the requirements of the geometric programming method:

$$P \le [P] \to 1.3 \cdot 10 \cdot C_{pz} K_{pz} f^{y_p} V^{n_p} d^{x_p} \le 6000;$$

$$T \ge T_{pr} \to V^{\frac{1}{m}-1} f^{\frac{y_V}{m}} d^{\frac{x_v}{m}} \le \frac{(C_V U_V)^{\frac{1}{m}}}{4.4};$$

$$T \ge t_m \to V^{\frac{1}{m}-1} f^{\frac{y_V}{m}-1} d^{\frac{x_V}{m}} \le \frac{(C_V U_V)^{\frac{1}{m}} \cdot 1000}{\pi DL}. \qquad (5.9)$$

Substituting the initial data of the problem, as well as the constants determined by the cutting conditions according to [5], we obtain the following system of constraints:

$$\begin{cases} 2.24 \cdot V^{-0.15} f^{0.9} \le 1; \\ 5.4 \cdot 10^{-13} \cdot V^{5.56} f^{0.83} \le 1; \\ 3.86 \cdot 10^{-12} \cdot V^{4.56} f^{-0.17} \le 1. \end{cases}$$

The formation of a mathematical model of the problem

In a direct formulation, the geometric programming problem has the form:

Minimize: $\eta_1(V, f) = 5118.26 + 398.42 V^{-0.113} f^{-0.217}$,

with restrictions: $2.24 \cdot V^{-0.15} f^{0.9} \le 1;$

$$5.4 \cdot 10^{-13} \cdot V^{5{,}56} f^{0{,}83} < 1;$$

$$3.86 \cdot 10^{-12} \cdot V^{4.56} f^{-0.17} \le 1.$$

In this problem, four posynomial terms n and two variables (V, f), which relates it to the problem of the first degree of difficulty ($n - m - 1 = 4 - 2 - 1 = 1$). Besides, the objective function contains a single posynomial term (dual weight $w_{01} = 1$). Let us pass to the dual problem of geometric programming with restrictions:

Maximize: $V(w) = C_{01}^{w_{01}} C_{11}^{w_{11}} C_{21}^{w_{21}} C_{31}^{w_{31}}$,

under restrictions: $w_{01} = 1;$

$$V: -0.113 w_{01} - 0.15 w_{11} + 5.56 w_{21} + 4.56 w_{31} = 0;$$
$$f: -0.217 w_{01} + 0.9 w_{11} + 0.83 w_{21} - 0.17 w_{31} = 0.$$

Using the program [66] of geometric programming developed by the authors, we solve this problem as a combination of two problems of zero degrees of difficulty. The first task determines the optimal set:

dual weights: $\{w_{01}^I = 1;\ w_{11}^I = 0.217;\ w_{21}^I = 0.226\};$

the minimum value of the variable part of the specific energy intensity: $\eta_1^{\mathrm{I}}(\mathrm{var}) = 0.23$ Дж / мм^3;

optimal cutting modes: $V_0^I = 162$ m/min; $f_0^I = 0953$ mm/rev.

The second task determines the optimal set:

dual weights: $\{w_{01}^{II} = 1;\ w_{11}^{II} = 0.247;\ w_{21}^{II} = 0.033\};$

minimum value: $\eta_1^{\mathrm{II}}(\mathrm{var}) = 0.2$ J / mm^3;

optimal values of cutting modes:

$$V_0^{II} = 319.15 \text{ m/min}; \quad f_0^{II} = 1.06 \text{ mm/rev}.$$

An analysis of the results shows that, despite the decrease in specific energy intensity in the second case, this set of modes cannot be used due to insufficient engine capacity of the machine 16K20F3.

The optimum of the criterion η_1 in this statement lies outside the boundaries of the feasible solutions range, therefore, to establish the optimal option for the functioning of the cutting process, the specific meaning of the criterion does not matter. In such cases, as noted, it is advisable to use the energy criterion.

To obtain reliable results, it is necessary to introduce parametric constraints, especially on the cutting speed: $V_{min} \le V \le V_{max}$, assigned according to the standards [20] or empirically. At the same time, in the first task, the obtained value of the cutting speed is 100% higher than the traditionally set values V on universal equipment, which approaches the recommendations [26]. The value of the cutting speed according to the standards [1] is $V = 73$ m/min.

6. OPTIMIZATION OF CUTTING MODES DURING TURNING, DRILLING AND MILLING

6.1. The technique of finding the optimal values of cutting modes

A typical linear programming problem is related to the search for the extremum (maximum or minimum) of the objective function in a linear form with constraints on the controlled variables, also expressed as linear dependencies. To optimize the cutting conditions, it is transformed into the task of finding such values of technological parameters that would deliver the maximum or minimum value of productivity, cost price, or some other criterion for the efficiency of the cutting process.

The formation of the objective function. In many technological studies, the goal is to maximize processing productivity, which reduces to minimizing the machine time t_m (time in cut) for an operation [8]:

$$t_m = \frac{L}{f \cdot n} = \frac{L\pi D}{1000Vf} \quad \rightarrow \quad \min .$$

Formation of a system of technical constraints. Constrains in the task of optimizing cutting modes are an analytical relationship between the technological conditions of processing (a given type of machine, parameters of the workpiece, detail, tool, and adjustment) and the operating modes (V, f, d) to be optimized. For single-pass machining, the cutting depth is assumed to be constant. When solving optimization problems, there should be no way beyond certain upper and lower permissible values of the characteristics for the machine, fixture, tool and workpiece.

Technical constraints during cutting can be divided into several groups [81]:

1) the ranges of possible values of the cutting modes elements, determined by the cutting ability of the tool when processing this material, the technical characteristics of the equipment, etc.;

2) the maximum permissible values of a number of the cutting process characteristics (temperature in the cutting zone, quality and accuracy of the machined surface, etc.), due to the requirements for the workpiece;

3) constrains on tool life, which takes into account the requirements for the periodicity of tool changes, associated with the peculiarity of the process and the organizational form of equipment maintenance;

4) the permissive values of cutting efforts permissible from the conditions of strength and rigidity of the machine-fixture-tool-detail system, taking into account the technical capabilities of the equipment and the required processing accuracy. These constraints apply to individual adjustment tools as well as to some of their autonomous groups. In the latter case, we are talking, as a rule, about the total efforts and capacities;

5) constrains due to available resources (material, temporary, etc.) on the processing process at a given time and in the required volume.

The number and meaning of the constraints depend on the specific features of the cutting process [82-84]. So, in rough turning, when large layers of metal are removed, the constraints depend on the life tool period, the strength of the tool material, the rigidity of the tool or its holder, the capacity of the machine main drive and the force, the permissible strength of the feed mechanism. When finishing turning, the main constrains will be the specified errors of the tolerance field and surface shape, as well as their relative position. Also, it is necessary to ensure a given surface roughness. Machining modes must comply with the passport data of the machine and not exceed their limit values.

Consider a system of technical constrains for rough turning.

The constraint 1 on the cutting capabilities of the tool establishes the relationship between the cutting speed V and the quantitative index of machinability V_T (V_T – cutting speed corresponding to a certain tool life):

$$V \le V_T . \qquad (6.1)$$

Here:

$$V = \frac{\pi D n}{1000}; \quad V_T = \frac{C_V K_V}{T^m d^{x_V} f^{y_V}},$$

where D – diameter of the workpiece, mm; C_V – coefficient taking into account the working conditions of tools; K_V – coefficient taking into account the difference between the specific working conditions of the tool and those taken as the basis; d – cutting depth, mm; f – feed, mm/rev; x_V, y_V, m – exponents reflecting the influence, respectively, of the cutting depth, feed and tool life T.

To implement the linear programming method and geometric interpretation of the cutting process model, it is necessary to solve inequality (2.1) for n and f, i.e. provide explicitly:

$$nf^{y_V} \le \frac{318 \cdot C_V K_V}{T^m t^{x_V} D}. \qquad (6.2)$$

Constrain 2 on the capacity of the electric motor for a drive of the machine main movement:

$$N_e \le N_e^m; \quad N_e^m = N_m \eta, \qquad (6.3)$$

here:

$$N_e = \frac{P_z V}{60 \cdot 1020} = \frac{C_{pz} K_{pz} f^{y_{Pz}} d^{x_{Pz}} \pi D n V^{n_{Pz}}}{60 \cdot 1020 \cdot 1000},$$

where: C_{Pz} – coefficient taking into account the influence of working conditions on the force and torques on the machine spindle, taken as a basis in the handbook; K_{Pz} – coefficient taking into account the difference between specific working conditions and those given in the handbook; n_{Pz}, x_{Pz}, y_{Pz} – degree indicators reflecting the influence of respectively the speed, cutting depth and feed on the tangential component of the cutting force P_Z; N_m – the power of the electric motor (the main driver of the machine according to the passport), kW; N_e, N_e^m – respectively, the effective capacity developed during the cutting process and the maximum capacity due to the capabilities of the machine, kW; η – transmission efficiency from the electric motor to the tool.

Solving equation (6.3) concerning n and f, we obtain

$$n^{n_{Pz}+1} f^{y_{Pz}} \le \frac{1000^{n_{Pz}}\, 612\cdot 10^4 \cdot N_e^m}{C_{P_Z} K_{P_Z} (\pi D)^{n+1} d^{x_{Pz}}}. \qquad (6.4)$$

Constrain 3 on the force, permissible strength of the weak link of the machine feed mechanism:

$$P_x \le [P_{mf}]; \quad P_x = C_{Px} K_{P_x} d^{x_{P_x}} S^{y_{P_x}} \cdot 9.81 \le [P_{mf}], \qquad (6.5)$$

where C_{Px}, K_{Px}, X_{Px}, Y_{Px} – are determined by handbook [30]; P_x – axial component of the cutting force, N; $[P_{mf}]$ – the maximum allowable force determined by the machine passport.

We represent (6.5) in the explicit form:

$$f^{y_{P_x}} \le \frac{[P_m]}{9.81 \cdot C_{P_x} d^{x_{P_x}} K_{Px}}. \qquad (6.6)$$

Constrain 4 is due to the strength of the tool holder. Considering the cutter as a cantilever loaded at the end with the tangential force P_z, we obtain the inequality:

$$M_b \le \frac{\sigma_b W_h}{K_s}, \qquad (6.7)$$

Here:

$$M_b = P_z \cdot l_c; \quad W_h = \frac{BH^2}{6}; \quad P_z = C_{pz} K_{pz} d^{x_{P_z}} f^{y_{Pz}} V^{n_{P_z}},$$

where M_b – bending moment, N·m; l_c – cutter overhang, mm; σ_b – tensile strength of the tool holder material during bending, MPa; W_h –section modulus of the holder, mm^3; K_s – safety factor; n_{Pz} – exponent characterizing the effect of velocity on the value of P_z.

Solving inequality (6.7) to f and n, we obtain:

$$n^{n_{P_z}} f^{y_{P_z}} \le \frac{\sigma_b W_h \cdot 1000^{n_{P_z}}}{C_{pz} K_{pz} d^{x_{Pz}} l_c (\pi D)^{n_{Pz}}}. \qquad (6.8)$$

Constrain 5 on the feed allowed by the strength of the cutting tip.

This constrain is typical for working with a tool with a cutting part made of cemented carbide tool, metal ceramics (cermet), diamond and cubic boron nitride:

$$f^{y_{P_z}} \le 34 \cdot c^{1.25} (\sin 60^0 / \sin \varphi)^{0.8} / C_{Pz} d^{(x_{P_z} - 0.77)} K_{pz}. \qquad (6.9)$$

Constrains 6–9 imposed by the machine kinematics establish the relationship of the calculated values of the rotational speed and feed with the allowable machine kinematics (maximum and minimum). These conditions can be written in the form of the following inequalities:

$$f \ge f_{\text{m min}}; \qquad (6.10)$$

$$f \le f_{\text{m max}}; \qquad (6.11)$$

$$\text{n} \ge \text{n}_{\text{m min}}; \qquad (6.12)$$

$$n \le n_{\text{m max}}. \qquad (6.13)$$

Development of a mathematical model of the cutting process

The mathematical model in the problem of optimizing cutting modes is the system of inequalities (6.2), (6.4), (6.6), (6.8), (6.9)–(6.13) and the equation of the objective function t_m (F_0) for roughing. To construct a mathematical model of the cutting process and use it to determine the optimal cutting modes for the basic principles of linear programming, it is necessary to convert all the inequalities of technical constraints and the objective function equation into linear forms by logarithm expressions. The transformation of inequalities into linear forms is followed by the example of technological constraint 1.

After the logarithm of inequality (6.2), we have:

$$\ln n + y_V \ln f \le \ln\left(\frac{318 \cdot C_V K_V}{T^m d^{X_V} D}\right). \tag{6.14}$$

We introduce the notation:

$$\ln n = x_1; \quad \ln f = x_2; \quad \ln\left(\frac{318 \cdot C_V K_V}{T^m d^{x_V} D}\right) = b_1,$$

and substituting them into expression (6.14), we obtain a linear form of inequality:

$$x_1 + y_V x_2 \le b_1.$$

By transforming inequalities (6.4), (6.6), (6.8), (6.9)-(6.13) and the equation of the objective function t_m in this way, we obtain a system of linear inequalities and a linear function for the case of rough turning in the form:

$$\left.\begin{array}{l}
x_1 + y_V x_2 \le b_1; \\
(n_{P_Z} + 1)x_1 + y_{P_Z} x_2 \le b_2; \\
y_{P_X} x_2 \le b_3; \\
n_{P_Z} x_1 + y_{P_Z} x_2 \le b_4; \\
y_{P_Z} x_2 \le b_5; \\
x_2 \ge b_6; \\
x_2 \le b_7; \\
x_1 \ge b_8; \\
x_1 \le b_9; \\
F_0 = (x_1 + x_2) \to \max.
\end{array}\right\} A.$$

The optimal values of n_{opt} and f_{opt} are calculated using the formulas:

$$n_{opt} = e^{x_1}; \tag{6.15}$$

$$f_{opt} = e^{x_2}. \tag{6.16}$$

The above mathematical model *A* describes the cutting process for the case of roughing machining regardless of the machine type and processing conditions. Model *A* can undergo some changes when changing the grade of the material of the cutting part of the tool. So, for cutting materials such as alloy tool steel and high-speed steel, constraint 5 is excluded from the constraint system. If the conditions for performing certain operations change, only the free terms b_1, b_2,...,b_9 and the coefficients y_V, y_{P_Z}, y_{P_X}, n_{P_Z} will be other.

To determine the optimal modes using the obtained model *A*, it is necessary to find positive values $\{x_1; x_2\}, \{x_1^*; x_2^*\}$ for which the linear form of the evaluation function would take on the greatest value. The solution of model *A* can be performed graphically and analytically.

To implement the graphical method, as the most effective and most visual one, the inequalities and the equations included in the model should be depicted by straight lines in the $n - f$ coordinate system. On the graph, it is necessary to draw lines in double logarithmic scales and indicate which side each line contains points corresponding to the permissible values of n and f (Fig. 6.1). When these lines intersect, a series of points is formed. If the system of constraints is not inconsistent, i.e. joint, the indicated set of points forms a convex closed polygon *ABCD*, which is a polygon of possible solutions.

The dependence $F_0 = (x_1 + x_2) \rightarrow \max$ of system A to be optimized is shown by a dashed straight line inclined at an angle of 45^0 to the coordinate axis. This linear function will be maximal when the dashed line takes the position at which the distance from the origin along the perpendicular to it will be the largest, which is possible in the case when the dashed line passes through point C of the solutions polygon. The coordinates of this point will give the optimal values of x_{1opt} and x_{2opt}, and the sum of the coordinates x_1+x_2 will be the largest.

Having found the coordinates x_{1opt} and x_{2opt} and using formulas (6.15) and (6.16), we determine the optimal values of the process control parameters n_{opt} and f_{opt}.

In the presence of a formed permissible solutions region (a convex polygon *ABCD*), it becomes possible to rationally select metal-cutting equipment based on the most complete consideration of its technical characteristics[34]. For this purpose, one should choose a machine from the existing nomenclature for which the ranges of the spindle speed and feed rate are as close as possible to the possible cutting conditions polygon without crossing it. In Fig. 6.1 to select the most rational equipment, it is necessary to apply various options for straight lines $\{n_i = n_{i\min}; n_i = n_{i\max}; f_i = f_{i\min}; f_i = f_{i\max}\}$ according to the passport data and select the best option (according to the minimum discrepancy with the solution polygon).

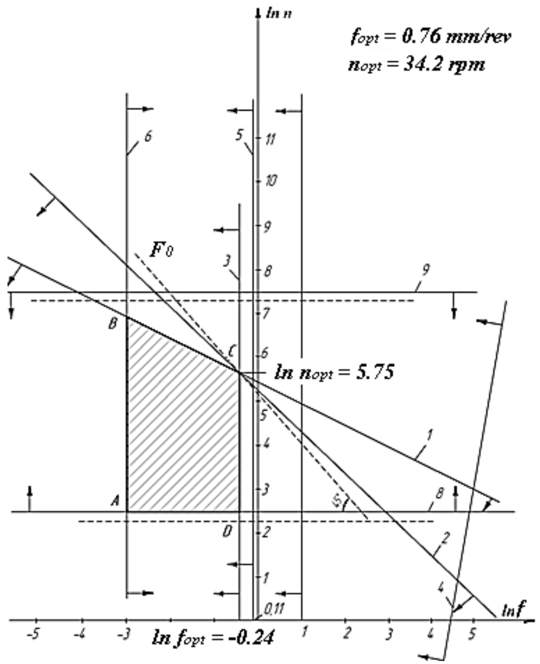

Fig. 6.1. The scheme of finding the optimal rough-cutting modes

6.2. Using the linear method programming to determine optimal cutting modes for shaft-type parts

Example 1. Initial data: the workpiece – shaft shown in Fig. 2.1, operation – rough turning, machine – screw-cutting lathe16K20F3. Type and size - rolled C45 (steel 45); σ_b = 598 MPa, D_1 = 200 mm; σ_{ben} = 200 MPa, the tool is a turning lathe tool with a mechanical fastening of a hexagonal plate made of DIN HS 123 (T15K6 hard alloy); $\varphi = 45^0$; $\varphi_1 = 10^0$; $\gamma = 10^0$; C = 5 mm; $b \times h$ = 25 × 25 mm; l_c = 25 mm; T = 60 min.

Conditions for the operation: the operation is performed in one pass; the workpiece is mounted in the centers. Cutting depth d = 3 mm.

It is required to determine the optimal values of the cutting speed and feed, providing the maximum value of the objective function – machining productivity.

Formation of a system of constraints

1. The constraint on the cutting capabilities of the tool is formed using the dependence (6.2). For this case of machining, the components of this dependence take the following values [30]:

$$C_V = 340; \quad x_V = 0.15; \quad y_V = 0.45; \quad m = 0,20;$$

$$K_V = K_{mV} K_{nV} K_w = 1.25 \cdot 1.0 \cdot 1.0 = 1.25;$$

$$K_{mV} = K_r \left(\frac{750}{\sigma_в}\right)^{n_V} = 1.0 \left(\frac{750}{598}\right)^{1.0} = 1,25;$$

$$n \cdot f^{0.45} \le \frac{318 \cdot 340 \cdot 1.25}{60^{0.20} \cdot 3^{0.15} \cdot 200} = 252.69.$$

2. The power constraint of the electric drive of the main movement of the machine. According to the data [30] of the external longitudinal turning of structural steels, the constituent dependencies (6.4) take the following values:

$$C_{pz} = 300; \; x_{pz} = 1; \; y_{pz} = 0.75; \; n = -0.15;$$

$$K_{Pz} = K_{mPz} K_{\varphi Pz} K_{\gamma Pz} K_{\lambda Pz} = 0.84; \quad K_{mPz} = \left(\frac{\sigma_b}{750}\right)^n = \left(\frac{598}{750}\right)^{0.75} = 0.84;$$

$$K_{\varphi Pz} = 1,0; \; K_{\gamma Pz} = 1,0; \; K_{\lambda Pz} = 1,0; \quad N_m = 10 \, kW; \; \eta = 0.8;$$

$$n^{0.85} f^{0.75} \le \frac{10 \cdot 0.8 \cdot 1000^{0.85} \cdot 1020 \cdot 60}{10 \cdot 300 \cdot 3^1 \cdot (3.14 \cdot 200)^{0.85} \cdot 0.84} = 96.13.$$

3. Constraint on the force allowed by the strength of the weak link of the machine feed mechanism (6.6). According to the data [30]:

$$C_{Px} = 339; \; x_{Px} = 1.0; \; y_{Px} = 0.5; \quad K_{Px} = K_{\varphi Px} K_{\gamma Px} K_{\lambda Px} = 1.0 \cdot 1.0 \cdot 0.65 = 0.65;$$

$$\left[P_{mf} \right] = 5884 \, N \text{ (Passport data of the machine);}$$

$$f^{0.5} \le \frac{5884}{10 \cdot 339 \cdot 3^{1.0} \cdot 0,65} = 0.89.$$

4. The constraint on the strength of the tool holder.

Substituting the previously determined data into dependence (6.8), we obtain:

$$n^{-0.15} S^{0.75} \le \frac{200 \cdot 3.3 \cdot 10^4 \cdot (1000)^{-0.15}}{10 \cdot 300 \cdot 0,84 \cdot 3^{1.0} \cdot 25 \cdot (3,14 \cdot 200)^{-0.15}} = 32.57.$$

5. The constraint on the strength of the cutting part tip for the cutter (when using high-speed tool steel is not taken into account in the calculations). For given values: C = 5 mm and φ = 45^0, substituting the previously determined data in (6.9), we obtain:

$$f^{0.75} \le \frac{34 \cdot 5^{1.25} \cdot \left(\frac{\sin 60^0}{\sin 45^0}\right)^{0.8}}{300 \cdot 3^{1.0-0.77} \cdot 0.84} = 0.92.$$

6. The constraint on the value of the minimum feed allowed by the kinematics of the machine: $f \ge 0.05$.

7. The constraint on the value of the maximum feed: $f \le 2.8$.

8. The constraint on the value of the minimum spindle speed of the machine: $n \ge 12.5$.

9. The constraint on the value of the maximum speed: $n \le 2000$.

Development of a mathematical model of the roughing cutting machining

For the case of turning roughing, model *A* should be taken as the basis, which is defined in the process of forming the listed nine restrictions and takes the following form:

$$\left.\begin{array}{l} x_1 + 0.45x_2 \le 5.53 \\ 0.85x_1 + 0.75x_2 \le 4.57 \\ 0.5x_2 \le -0.12 \\ -0.15x_1 + 0.75x_2 \le 3.48 \\ 0.75x_2 \le -0.08 \\ x_2 \ge -3.0 \\ x_2 \le 1.03 \\ x_1 \ge 2.53 \\ x_1 \le 7.6 \\ F_0 = (x_1 + x_2) \to \max \end{array}\right\} A'.$$

Graphical interpretation and definition of optimal cutting conditions

To find n_{opt} and f_{opt}, it is graphically necessary to construct a polygon of possible solutions to the constraint system included in A' Fig. 6.1 in the double logarithmic scales, the direct inequalities of the system A' are shown and the region of possible solutions *ABCD* of this system corresponding to the meaning of its inequalities is depicted. The boundary lines *AB*, *BC*, *CD*, and *DA*, intersecting, form a polygon, each of the points inside which satisfies the inequalities of all boundary lines of the system A' involved in its formation.

To find the optimal combination of elements $\{n, f\}$, it is necessary to determine at which point of the desired polygon *ABCD* the linear function of two variables $F_0 = (x_1 + x_2)$ will take the maximum value. For

this, the line F_0 must be moved parallel to itself, in the direction from the origin. In fig. 6.1 at the vertex of the polygon *C*, the objective function takes the greatest value. Therefore, the vertex *C* is the optimum point, and its coordinates x_{1opt} and x_{1opt} are the optimal solutions to model A'.

Thus, the optimal combination of elements of the cutting mode for the case of roughing, processing is possible at f = 0.76 mm/rev, n = 314.0 rpm, V = 197.2 m/min. In accordance with the machine data f = 0.76 mm/rev, n = 315.0 rpm, V = 196.0 m/min. These modes will correspond to the value of the main technological time (time in the cut) for processing:

$$t_c' = \frac{300}{0.76 \cdot 315} = 1.24 \text{ min}.$$

The constructed polygon of possible solutions serves as the basis for choosing the most rational equipment for the processable part. In this case, it is necessary to choose a machine whose characteristics are closest to the polygon *ABCD*. Such a machine is 16K25 (n = 12.5 ... 1600; f = 0.05 ... 2.8; N_m = 11 kW). Changing the model of the machine should be accompanied by a check on the effective capacity of the machine and the dimensions of the working area.

6.3. Optimization of cutting modes when drilling and milling

As a method of machining, drilling has several features:

1. Variable cutting speed along the length of the cutting edge.
2. Changing front and rear corners along the length of the cutting edge.
3. The presence of a transverse edge or jumper, which impedes the cutting process.
4. Difficult chip removal.

5. Low rigidity of the tool and the technological system as a whole.

In the process of drilling structural materials, the tool is subject to significant axial compressive forces P_a and torque M_t. These factors limit the assigned cutting modes and are the main ones when considering a set of constraints in the optimization problem.

Formation of technical constraints

1. The constraint on the cutting capabilities (tool life), which establishes the relationship between the cutting speed V and the machinability index V_T (cutting speed corresponding to a certain tool life) V_T.

Using the known dependencies [16]:

$$V = \frac{\pi D n}{1000}; \quad V_T = \frac{C_v K_v D^{q_v}}{T^m f^{y_v}},$$

where C_v, K_v – constants that take into account the operating conditions of the tool in the reference and specific versions; D – hole diameter, mm; T – tool life period, min; f – feed, mm/rev; n – rotation frequency, rpm; q_v, m, y_v – exponents reflecting the influence of diameter, tool life and feed on cutting speed.

We bring this constraint to an explicit form:

$$n f^{y_v} \le \frac{318 D^{(q_v - 1)} C_v K_v}{T^m}. \quad (6.1)$$

2. The power constraint of the machine M_{tm}. It binds the torque that occurs during drilling M_t and affects the spindle of the machine:

$$M_t = 10 \cdot C_m f^{y_m} D^{q_m} K_m;$$

$$M_{tm} = \frac{975 \cdot 10^3 N_m \cdot \eta}{n},$$

where C_m – coefficient taking into account the influence of the processing conditions; K_m – the general correction factor, taking into account the actual processing conditions and depending only on the workpiece materials; y_m, q_m – indicators of degrees with variables; N_m – capacity of the main drivers of an electric motor, kW; η – coefficient of performance.

Equating the right parts of the last two dependencies and making the corresponding transformations, we obtain:

$$n f^{y_m} \le \frac{975 \cdot 10^3 N_m \cdot \eta}{C_m D^{q_m} K_m}. \quad (6.2)$$

The constraint on M_t guarantees the integrity of the drill in the presence of stresses in its material.

3. Strength constraint of the machine feed mechanism. To implement the cutting process, the condition ist must perform:

$$P_a = 10 C_p D^{q_p} f^{y_p} K_p \le [P_m],$$

where P_a, $[P_m]$ – respectively, the axial and maximum cutting forces allowed by the feed mechanism of the machine N; C_p, q_p, y_p, K_p – determined by reference.

4. Tool strength constraint. This constraint is explicitly presented below:

$$f^{y_p} \le \frac{[P]}{10 C_p D^{q_p} K_p}. \quad (6.3)$$

4 The constraint on the strength of the tool. The condition for the drilling strength is expressed by the dependence

$$\tau_s = \frac{1.73 M_t}{W} = \frac{1.73 \cdot 10 \cdot C_m f^{y_m} \cdot D^{q_m} \cdot K_p}{0.02 D^3} \le \frac{\sigma_b}{K_{sf}},$$

where τ_s – the total stress equal to the sum of the normal stress (0.73 τ_s) of the force P_a and shear stress of M_t, MPa; σ_b – temporary resistance of the drill material to rupture, MPa; K_{sf} = 1.5…2.0 – safety factor; W – the moment of resistance of the drill section, mm^3.

After the corresponding transformations, we obtain:

$$f^{y_m} \le \frac{\sigma_b \cdot 0.02 \cdot D^3}{1.73 \cdot 10 \cdot C_m \cdot K_{sf} \cdot D^{q_m} \cdot K_p}. \tag{6.4}$$

5. The constraint on the rigidity of the cutting tool.

The rigidity of the tool affects the accuracy of processing, while the axial force should not exceed the permissible axial force [P_a] on the rigidity of the drill:

$$10 C_p D^{q_p} f^{y_p} K_p \le \frac{K_s E I}{L_{od}^2},$$

K_s – stability coefficient, $K_s \approx 2.46$; E – elastic modulus of the drill material, MPa; $I = 0.039 \cdot D^4$ – the moment of inertia of the drill, mm^4; L_{od} – the drill overhang length.

In explicit form, this restriction is written

$$f^{y_p} \le \frac{K_s E I}{L_{od}^2 \cdot 10 C_p \cdot D^{q_p} \cdot K_p}. \tag{6.5}$$

The fulfillment of constraint 5 guarantees the integrity of the drill in the event of a possible loss of longitudinal stability.

The constraints imposed by the machine kinematics, establish the relationship of the calculated values of the rotation frequency and feed with the allowable kinematics of the machine. These conditions can be written as the following inequalities:

$$n_{m\ \min} \le n \le n_{m\ \max}; \quad f_{m\ \min} \le f \le f_{m\ \max}.$$

Setting the objective function. For a large number of production situations when the values of economic tool life periods are used in the calculations, the least machine time t_m or specific processing costs C_s should be chosen as the objective function [85, 86].

$$t_m = \frac{L}{f \cdot n} = \frac{l + l_1 + l_2}{f \cdot n}; \tag{6.7}$$

$$C_s = \frac{a_1}{q} \left(\frac{1 + \frac{a_n}{a_1} \cdot \frac{1}{T}}{n \cdot f} \right), \tag{6.8}$$

where L – processing length, mm; l – hole length, mm; l_1, l_2 – the values of the plunge and overrun cutting of the tool, determined analytically and by reference data: $l_1 = \frac{D}{2} ctg\varphi$, $l_2 = 1...3$, mm; a_1 – the cost of the machine tool minute of the basic equipment (cent/min); q –

cutting parameter ($q = \pi D^2/4$ – an area of the removed metal); $a_t = a_1 t_{ch} + a_2 t_{rg} z/(z+1) + \mathrm{b}/(z+1)$ – tool costs for durability period, cent; a_2 – the cost of machine tools for grinding equipment, cent/min; b - the cost of a new tool, taking into account transport costs and the implementation of waste, cent; t_{ch} – tool change time, min; t_{gr} - tool regrinding time, min; z – the number of regrinding of the tool until it is completely wearing out.

Mathematical model cutting process development

The mathematical model in the problem of optimizing cutting modes during drilling is jointly the system of inequalities (6.1) – (6.6) and the objective function equation (6.7) or (6.8). Depending on the method used, the original model (6.1) – (6.7), (6.8) undergoes certain transformations. The statements of the optimization problem that are oriented to the method of linear programming are considered below.

Linear programming. The transformation of the original model is carried out by logarithm expressions of constraints (6.1) – (6.6) and the objective function (6.7) and obtain the corresponding linear forms.

After the logarithm of inequality (6.1), we have (an example of reduction)

$$\ln n + y_v \ln f \le \ln\left(\frac{318 \cdot C_v K_v D^{(q-1)}}{T^m}\right). \qquad (6.9)$$

We introduce the notation $\ln n = x_1$; $\ln f = x_2$; $\ln\left(\frac{318 \cdot C_v K_v D^{(q-1)}}{T^m}\right) = b_1$ and substitute them into expression (6.9). As a result of the substitution, we obtain the linear form of the inequality

$$x_1 + y_v x_2 \le b_1.$$

Transforming inequalities (6.2) – (6.7) in this way, we obtain a system of linear inequalities and a linear function for the case of a drilling operation in the form

$$\left.\begin{array}{l} x_1 + y_v x_2 \le b_1 \\ x_1 + y_m x_2 \le b_2 \\ \mathrm{y_p} x_2 \le \mathrm{b}_3 \\ y_m x_2 \le b_4 \\ \mathrm{y_p} x_2 \le b_5 \\ x_2 \ge b_6 \\ x_2 \le b_7 \\ x_1 \ge b_8 \\ x_1 \le b_9 \\ F_o = (x_1 + x_2) \to \max \end{array}\right\} A$$

The optimal values n_0 and f_0 are calculated by the formulas

$$n_o = e^{x_1}; \quad f_o = e^{x_2}. \qquad (6.10)$$

The above form of mathematical model A provides a description of the cutting process for the drilling case, regardless of the machine type and processing conditions. If the conditions for performing certain operations change, only the free terms $b_1, b_1, \ldots, b_9$. and the coefficients y_1, y_m, y_p will be other. To determine the optimal modes using model A, it is necessary to find positive values $\{x_1, x_2\}$ at which the linear form of the objective function (6.7) would take on the greatest value.

Optimization of cutting conditions during drilling.
Implementation examples

Example 1. Linear programming. On a vertical-boring machine 2H135, process a through hole with a diameter D = 12H13 to a length l = 55 mm. The workpiece material – Ti Grade 6 (VT5 titanium-based alloy σ_b = 900 MPa); type of workpiece – hot-rolled steel. Tool – twist drill with double sharpening. Geometric parameters: $2\varphi = 140^0$; $\alpha = 12^0$; $2\varphi_0 = 90^0$; $w = 30^0$; reverse taper 0.1 ... 0.15 mm per 100 mm of length.

Passport data of the vertical drilling machine 2H135:

- the largest drilling diameter D = 35 mm;
- spindle speed, min^{-1}: n = 31 ... 1400;
- spindle feed, mm/rev: f = 0.1 ... 1.6;
- the greatest feed force: P_{mf} = 15000 N;
- capacity of the main drive electric motor: N_m = 4.5 kW;
- coefficient of performance: η = 0.8.

Formation of constraints system

1. *The constraint on cutting capabilities*. Using tool life dependence (6.1) and reference data [87]: $\{C_v = 2.8$; q = 0,7; y_v = 0.6; m = *0.5;* $K_v = 1\}$, we get:

$$n \cdot f^{0.6} \le \frac{318 \cdot 12^{-0.3} \cdot 2.8}{12^{0.5}} = 122.11.$$

The recommended tool life period T for the case of hole machining D = 12 mm with a cemented carbide tool is T = 12 min [87].

1. *The power constraint of the main movement drive of the machine.*

According to [87], the components of inequality (6.2) assume the following values: $\{C_m = 60;\ q_m = 1.9;\ y_m = 0.8;\ K_p = 1\}$. Group VII materials (titanium-based alloys) are characterized by the effect of cutting speed on torque. With this in mind, this constraint takes the form:

$$n^{0.85} \cdot f^{0.8} \le \frac{975 \cdot 10^3 \cdot 4.5 \cdot 0.8 \cdot (1000)^{-0.15}}{(\pi \cdot 12)^{-0.15} \cdot 60 \cdot 12^{1.9}} = 314.31.$$

3. *Strength constraint of the machine feed mechanism*. Given the expression (6.3) and reference data [87]: $\{C_p = 850;\ q_p = 1;\ y_p = 0.7\}$ this restriction takes the form

$$f^{0.7} \le \frac{15000}{850 \cdot 12} = 1.47.$$

4. *The constraint on the strength of the tool*. Given the expression (6.4) and reference data [87]:

$$n^{-0.15} \cdot f^{0.8} \le \frac{900 \cdot 0,02 \cdot 12^{1.25} \cdot 318^{-0.15}}{1.73 \cdot 10 \cdot 60 \cdot 1.5} = 0.108.$$

5. *The constraint on the rigidity of the cutting tool*. Given expression (6.5) and data [87]:

$$f^{0.7} \le \frac{2.46 \cdot 220000 \cdot 0.039 \cdot 12^4}{120^2 \cdot 12 \cdot 850} = 2.98.$$

6. *Parametric constraints on the passport of the machine [30]:*

$$31 \le n \le 1400; \quad 0,1 \le f \le 1,6.$$

The formation of the objective function

As the objective function, we choose the machine time t_m spent on drilling a through-hole 55 mm long:

$$t_m = \frac{55 + \left(\frac{12}{2} \cdot ctg70 + 3\right)}{S \cdot n} = \frac{60}{f \cdot n}.$$

Mathematical model

For the case of through-hole drilling, model *A* takes the following form:

$$\left.\begin{array}{l} x_1 + 0.6x_2 \le 4.8 \\ 0.85x_1 + 0.8x_2 \le 5.75 \\ 0.7x_2 \le 0.39 \\ -0.15x_1 + 0.8x_2 \le -2.23 \\ 0.7x_2 \le 1.09 \\ x_1 \le 7.24 \\ x_1 \ge 3.43 \\ x_2 \le 0.47 \\ x_2 \ge -2.3 \\ F_o = x_1 + x_2 \to \max \end{array}\right\} A'.$$

Graphical interpretation and definition of optimal cutting modes

To graphically find $\{n_o \text{ and } f_o\}$, it is necessary to construct a polygon of possible solutions to the system of constraints included in *A′*.

In Fig. 6.2 the direct inequalities of the system *A′* are shown and the area of possible solutions *ABC* of this system is selected [88-91]. The boundary lines *AB, AC* and *BC* intersecting each other, form a polygon, each of the points inside which satisfies the inequalities of all boundary lines of the system *A* involved in its formation. To find the optimal combination of elements, it is necessary to determine at which point of the required *ABC* polygon the linear function of two variables $F_o = (x_1 + x_2)$ will take the maximum value. To do this, you need to move the line F_0 parallel to itself in the direction from the origin. At the vertex of the polygon (in this case, the triangle) *C*, the objective function takes on the greatest value.

Therefore, vertex *C* is the optimum point, and its coordinates $\{n_o, f_o\}$ are the optimal solution to model *A′*.

Thus, the optimal combination of cutting mode elements for the case of drilling a through-hole in a Ti Grade 6 (VT5 titanium alloy) detail is VK8 carbide tool: $f_0 = 0.18$ mm/min; $n_0 = 337$ rpm; $V_0 = 12.7$ m/min; $f_m = 121.5$ mm/min; $t_m = \frac{55+2+3}{337 \cdot 0.18} = 0.99$ min.

Example 2. *Geometric programming*. Determine the optimal cutting modes during drilling (source data are borrowed from example 1).

Formation of a system of constraints

Using the obtained polygon of feasible solutions (Fig. 6.2), we distinguish two active constraints:

1 – constraint on cutting capabilities:

$$n \cdot f^{0.6} \le 122.11.$$

4 – constraint on the strength of the tool:

$$n^{-0.75} \cdot f^{0.8} \le 0.108.$$

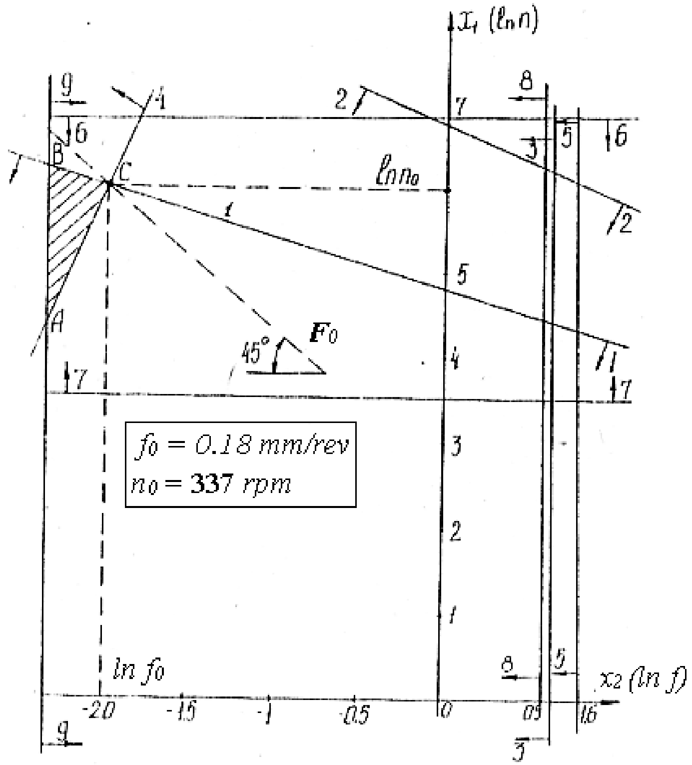

Fig. 6.2. The scheme of finding the optimal drilling cutting modes

We bring the system of active constraints to the standard form in the GP:

$$0.0082 \cdot n \cdot f^{0.6} \le 1;$$

$$9.26 \cdot n^{-0.15} \cdot f^{0.8} \le 1.$$

Assignment the objective function

As the objective function, we consider the specific cost of processing C_s [62]. According to the data of [1], the components of the dependence (6.8) take the following values: min; a_1 = 6 cent/min; a_2 = 12 cent/min; t_{ch} = 0.7 min; t_{pa} =2.0 min; b = 30 cent; z = 15; $q = \pi \cdot 12^2 / 4 = 113$ mm^2.

Objective function C_s takes the form:

$$C_s = \frac{0.06}{113} n^{-1} f^{-1} + \frac{0.06 \cdot 0.7 + (0.012 \cdot 2 \cdot 15)/16 + 0.3/16}{113} \cdot T^{-1} \cdot n^{-1} \cdot f^{-1} =$$
$$= 0.0005 n^{-1} f^{-1} + 0.007 T^{-1} n^{-1} f^{-1}.$$

Here T is determined from the extended Taylor equation [30]:

$$T = \frac{(C_v K_v)^{1/m} \cdot D^{q/m}}{f^{y/m} \cdot V^{1/m}} = \frac{(2 \cdot 8 \cdot 1)^{1/0.5} \cdot D^{0.7/0.5}}{f^{0.6/0.5} \cdot V^{1/0.5}} = 254.17 \cdot f^{-1.2} \cdot V^{-2}.$$

After substituting the expression for T into the equation of the objective function, we obtain:

$$C_s = 0.0005 \cdot n^{-1} \cdot f^{-1} + 3.91 \cdot 10^{-9} \cdot f^{0.2} \cdot n.$$

Mathematical model

A direct statement of the GP problem during drilling is represented as follows:

Minimize:

$$g_o(f, n) = C_s = 0.0005 \cdot n^{-1} \cdot f^{-1} + 3.91 \cdot 10^{-9} \cdot n \cdot f^{0,2},$$

under constraints:

$$0.0082 \cdot n \cdot f^{0,6} \le 1;$$

$$9.26 \cdot n^{-0.15} \cdot f^{0.8} \le 1.$$

This task is the GP task of the first degree of difficulty. To solve it, we use the Wild method [34] and represent it as a zero-degree GP problem by discarding the second term of the objective function associated with the cost of tool changing.

Direct GP statement of zero degrees of difficulty:

Minimize:

$$C_s^1 = 0.0005 \cdot n^{-1} f^{-1}, \qquad (6.18)$$

under constraints:

$$0.0082 \cdot n \cdot f^{0.6} \le 1; \qquad (6.19)$$
$$9.26 \cdot n^{-0.15} \cdot f^{0.8} \le 1.$$

The dual GP statement of zero degrees of difficulty:

maximize:

$$V(w) = \left(\frac{C_{01}}{w_{01}}\right)^{w_{01}} \cdot C_{11}^{w_{11}} \cdot C_{21}^{w_{21}},$$

under the constraints:

$$w_{01} = 1;$$
$$-w_{01} + w_{11} - 0.15 w_{21} = 0;$$
$$-w_{01} + 0.6 w_{11} + 0.8 w_{21} = 0.$$

The solution of the last system of linear equations allows us to uniquely determine the dual weights w_i:

$$\{w_{01} = 1;\ w_{11} = 1.7;\ w_{21} = 0.45\};$$

The value of the objective function is:

$$V(w) = 0.005 \cdot 0.0082^{1.07} \bullet 9.26^{0.45} = 8.03 \cdot 10^{-6}, \text{ EUR/mm}^3.$$

From the conditions of invariance, we determine the optimal values of the cutting modes:

$$0.0082 \cdot n \cdot f^{0.6} = 1;$$
$$9.26 \cdot n^{-0.15} f^{0.8} = 1.$$

As a result of solving the last system of equations, we obtain:

$$n_0 = 337 \text{ rpm};\ f_0 = 0.18 \text{ mm/rev};\ V_0 = 12.7 \text{ m/min}.$$

By the partial invariance method [34], we carry out the following sequence of procedures:

1. We will formulate a model of the GP problem of the first degree of difficulty by introducing an additional term related to the cost of tool changing in the objective function (6.18):

$$C_s^{II} = 0.0005 \cdot n^{-1} f^{-1} + 3.91 \cdot 10^{-9} f^{0.2} n. \qquad (6.20)$$

The constraint for this option is stated above (6.19).

2. We define the lower bound of the objective function C_s^{II} in the statement (6.20), (6.19). In this case, the optimal values of the variables obtained as a result of solving the original problem of zero degrees of difficulty should be used. The cost of processing in this case:

$$C_s^{II} = 0.0005 \cdot 337^{-1} \cdot 0.18^{-1} + 3.91 \cdot 10^{-9} \cdot 337 \cdot 0.18^{0.2} =$$
$$= 8.24 \cdot 10^{-6} + 0.935 \cdot 10^{-6} = 9.18 \cdot 10^{-6}.$$

3. We calculate the basic weights of the members of the objective function by dividing the components of C_s by their sum:

$$w_{01}^{I}=\frac{8.24}{9.18}=0.898;\ w_{02}^{I}=\frac{0.935}{9.18}=0.102.$$

The dual weights of the constraints $\{w_1,\ w_2\}$ remain unchanged. A new set of dual scales is

$$w_{01}=0.898;\ w_{02}=0.102;\ w_{11}=1.07;\ w_{21}=0.45.$$

4. We compose a system of linear equations in the GP dual statement, including the conditions of orthogonality and normalization using the system (6.20), (6.19).

$$\begin{aligned} f:&-w_{01}+0.2w_{02}+0.6w_{11}+0.8w_{21}=0;\\ n:&-w_{01}+w_{02}+w_{11}-0.15w_{21}=0;\\ &w_{01}+w_{02}=1. \end{aligned} \quad (6.21)$$

5. We choose the dominant terms in the system (6.21), which does not have a unique solution. These should include the most significant members in the optimal project, and their number should exceed the number of project variables (*f, n*) by one unit. For this example, such members will be the first ($w_{01}=0.898$), the third ($w_{11}=1.07$) and the fourth ($w_{21}=0.45$).

6. We solve the system of linear equations (6.21) to the dominant dual variables, i.e. variables corresponding to dominant members. In the case under consideration, linear dependences on w_{02} are obtained:

$$w_{01}=1-w_{02};\ w_{11}=1{,}068-2w_{02};w_{21}=0{,}45w_{02}.$$

For all w_i to be positive, w_{02} it must be less than 0.5338. For this problem of the first degree of difficulty (6.20), (6.19) it is easy to calculate the dual function for various values of the only redundant dual variable (in our example, w_{02} – the weight of the objective function term associated with the tool change). The same dual function can be calculated for the values of the weights w_i giving the largest lower bound, and thereby determine the minimum of the "direct" function in the "direct" formulation.

7. Let us express the dual function in a general form as a function of the dual weight w_{02}:

$$V\left(w_{02}\right)=\frac{\left(\frac{0.0005}{1-w_{02}}\right)\cdot\left(\frac{1-w_{02}}{0.0005}\right)\cdot 3.91\cdot 0.0082^{1.068}\cdot 9.26^{0.45}}{w_{02}^{w_{02}}\cdot 0.0082^{2w_{02}}\cdot 10^{9w_{02}}}=$$
$$=\frac{8.04\cdot 10^{-6}\cdot\left(1-w_{02}\right)^{w_{02}-1}\cdot 0.12^{w_{02}}}{w_{02}^{w_{02}}}. \quad (6.22)$$

8. We use the dichotomy method [38, 68] to determine the maximum of the function $V(w_{02})$. Using the developed program for optimizing unimodal functions, the following result was obtained:

$$w_{02}=0.019;\ V\left(w_{02}\right)=C_s=9.003\cdot 10^{-6}\ \text{EUR/mm}^3.$$

9. Determine the optimal cutting modes that minimize the specific processing cost, based on the conditions of invariance:

$$\begin{aligned} &0.005\cdot n^{-1}\cdot f^{-1}=9.003\cdot 10^{-6}\cdot 0.8981;\\ &0.0032\cdot n\cdot f^{0.6}=1. \end{aligned} \quad (6.23)$$

As a result of solving the system of nonlinear equations (6.23), we obtain:

$$n_0 = 337.52 \text{ rpm}; \quad f_0 = 0.183 \text{ mm/rev}; \; V_0 = 12.72 \text{ m/min};$$

$$f_m = 61.83 \text{ mm/min}; \; C_{s0} = 9.003 \cdot 10^{-6} \text{ EUR/mm}^3$$

Example 3. The method of Lagrange multipliers. Determine the optimal cutting modes during drilling (source data are borrowed from example 1).

Let us consider the use of the Lagrange multiplier method for solving the optimization problems formulated above.

Drilling

The objective function and constraints on cutting tool capabilities and tool strength respectively are of the form:

$$C = 0.005 \cdot n^{-1} S^{-1} + 3.91 \cdot 10^{-9} \cdot nf^{0.2} = \alpha_1 n^{-1} f^{-1} + \alpha_2 nf^{0.2}$$

$$nf^{0,2} \le 122.11 = \beta_1;$$

$$n^{-0,15} S^{0,8} \le 0,108 = \beta_2.$$

We compose the Lagrange function:

$$L = C + \lambda_1 \cdot \left(\beta_1 - nf^{0.6}\right) + \lambda_2 \cdot \left(\beta_2 - n^{-0.15} S^{0.8}\right),$$

where λ_1 λ_2 – Lagrange multipliers.

Next, we have

$$\begin{cases} \dfrac{\partial L}{\partial n} = -\alpha_1 n^{-2} f^{-1} + \alpha_2 f^{0.2} - \lambda_1 f^{0.6} + 0.15 \cdot \lambda_2 n^{-1.15} f^{0.8} = 0; \\ \dfrac{\partial L}{\partial f} = -\alpha_1 n^{-1} f^{-2} + 0,2 \cdot \alpha_2 nf^{-0.8} - 0,6 \cdot \lambda_1 f^{-0.4} - 0.8 \cdot \lambda_2 n^{-0.15} f^{-0.2} = 0; \\ \dfrac{\partial L}{\partial \lambda_1} = \beta_1 - nf^{0.6} = 0; \\ \dfrac{\partial L}{\partial \lambda_2} = \beta_2 - n^{-0.15} f^{0.8} = 0. \end{cases}$$

Let $\lambda_1 \ne 0;$; $\lambda_1 = 0.$ Then we get:

$$\frac{-\alpha_1}{nf} + \alpha_2 nf^{0.2} - \lambda_1 \beta_1 = 0;$$

$$\frac{-\alpha_1}{nf} + 0.2 \cdot \alpha_2 nf^{0.2} - 0.6 \cdot \lambda_1 \beta_1 = 0;$$

$$\frac{-0.4 \cdot \alpha_1}{nf} + 0.4 \cdot \alpha_2 nf^{0.2} = 0 \rightarrow \begin{Bmatrix} \alpha_2 n^2 f^{1.2} = \alpha_1 \\ nf^{0.6} = \beta_1 \end{Bmatrix} \rightarrow \begin{Bmatrix} f^{0.84} = \dfrac{\alpha_1}{\alpha_2 \beta_1^2} \\ n = \beta_1 f^{-0.6} \end{Bmatrix}.$$

Thus, one of the possible solutions has the form:

$$\begin{cases} f = 8.58^{1.19} = 12.91; \\ n = 122.11 \cdot 12.91^{-0.6} = 26.31. \end{cases}$$

We check this solution for admissibility. To do this, substitute the found value in the second constraint:

$$26.31^{-0.15} \cdot 12.91^{0.8} = 4.74 > 0.108,$$

i.e. the constraint is not fulfilled and the solution is not in the range of acceptable values.

Let $\lambda_1 = 0;\ \lambda_2 \neq 0.$ Then we get:

$$\begin{cases} \dfrac{-\alpha_1}{nf} + \alpha_2 n f^{0.2} + 0.15 \cdot \lambda_2 \beta_2 = 0; \\ \dfrac{-\alpha_1}{nf} + 0.2 \cdot \alpha_2 n f^{0.2} - 0.8 \cdot \lambda_2 \beta_2 = 0; \end{cases}$$

$$-9.5 \cdot \alpha_1 n^{-1} f^{-1} + 8.31 \cdot \alpha_2 n f^{0.2} = 0 \to \left\{ \begin{matrix} 8.31 \cdot \alpha_2 n^2 f^{1.2} = 9.5 \cdot \alpha_1 \\ n^{-0.15} f^{0.8} = \beta_2 \end{matrix} \right\} \to$$

$$\to \left\{ \begin{matrix} n = \dfrac{1.07 \cdot \left(\dfrac{\alpha_1}{\alpha_2} \right)^{0.5}}{f^{0.6}}; \\ f^{0.89} = \dfrac{\beta_2}{1.07^{-0.15} \cdot \left(\dfrac{\alpha_1}{\alpha_2} \right)^{-0.075}} \end{matrix} \right\}.$$

The second possible solution has the form

$$f = \left(\frac{0.108}{1.07^{-0.15} \cdot \left(\dfrac{0.0005}{3.91 \cdot 10^{-9}} \right)^{-0.075}} \right)^{\frac{1}{0.89}} = \left(\frac{0.26}{0.99} \right)^{1.12} = 0.22;$$

$$n = \frac{389.78}{0.22^{0.6}} = 966.86.$$

Substitute the found values into the first constraint:

$$966.86 \cdot 0.22^{1.12} = 177.36 > 122.11,$$

i.e. the constraint is not met.

Let $\lambda_1 \neq 0;\ \lambda_2 \neq 0$. Then the solution of the general problem is the solution of the system of equations

$$\left\{ \begin{matrix} n f^{0.6} = \beta_1 \\ n^{-0.15} f^{0.8} = \beta_2 \end{matrix} \right\} \to \begin{cases} n = \dfrac{\beta_1}{f^{0.6}} \\ f^{0.89} = \dfrac{\beta_2}{\beta_1^{-0.15}}. \end{cases}$$

The third possible solution is

$$f = \left(\frac{0.108}{122.11^{-0.15}} \right)^{1.12} = 0.185;$$

$$n = \frac{122.11}{0.185^{0.6}} = 336.08.$$

This solution satisfies both constraints and is the desired solution to the problem.

Optimization of cutting conditions during milling

Milling is a method of multi-blade machining of groove planes, shaped surfaces, bodies of revolution, as well as the manufacture of splines and cutting the workpiece, which allows us to obtain a surface roughness of R_z 40 ... R_a 2.5 and a quality grade of 12 ... 9. The features of the cutting process during milling include:

- the simultaneous presence in the process of cutting several teeth. The larger this number, the lower the intensity of the vibrations during cutting;
- cyclic stresses on the tooth in the mode: load – rest;
- periodically repeated the tooth entrance into the metal, leading to impact loads on the cutting edges, as well as in the presence of a rounding radius, the occurrence of a certain sliding period without cutting;
- variability of the cutting edge load for one cutting cycle, due to the variable size of the area of the cut layer.

The milling process takes place in the condition of a specific technological system and is described by certain permissible characteristics of cutting capacity, levels of tool loads, tool life and reliability of operation. The composition of constraints on possible sets of cutting modes includes the following set.

Technical constraints

1. Constraint on the cutting capabilities of the tool:

$$nf^{y_v} \le \frac{318 \cdot C_v D_m^{q_v - 1} K_v}{T^m d^{x_v} z^{u_v} B^{r_v}}, \qquad (6.24)$$

where D_m – mill cutter diameter, mm; z – cutter number of teeth; B – the width of the milling, mm

2. Machine capacity constraint:

$$nf^{y_p} \le \frac{975 \cdot 10^3 N_m \eta K_{cz}}{C_p d^{x_p} z^{u_p} B^{r_p} K_p D_m^{1-q_p}}. \qquad (6.25)$$

3. Strength constraint of the machine feed mechanism. To implement the milling process, the condition must be carried out:

$$P_f \le [P_m],$$

where P_f – force that the machine feed mechanism overcomes. In the general case:

$$P_f = P_c + F,$$

where P_c – a force generated in the cutting process; F – friction force in the machine guides.

For the milling process, the force P_f is determined by the following dependence (taking into account relations [16] and the value of the friction coefficient $f_c = 0.1$):

$$P_f = P_c + f_c\left(P_z + P_y + P_x\right) = 0.7 \cdot P_z + 0.1 \cdot \left(P_z + 0.4 \cdot P_z + 0.5 P_z\right) = 0.89 \cdot P_z.$$

Explicitly, the feed constraint takes the form:

$$f^{y_p} \le \frac{P_f}{C_p d^{x_p} z^{u_p} B^{r_p} K_p D_m^{1-q_p}}. \qquad (6.26)$$

Constraints on the strength of the cutter, the strength of the cemented-carbide tip, the rigidity of the cutting tool and the workpiece, as a rule, are not taken into account when solving optimization problems [92].

4. Parametric constraints imposed by the kinematics of the machine:

$$n_{m.\min} \le n \le n_{m.\max};\; f_{m.\min} \le f \le f_{m.\max}.$$

Objective functions during milling

1. The main time in cut t_c for an operation performed on milling machines is calculated by the formula

$$t_c = \frac{L}{f_m} i = \frac{l + l_1 + l_2}{f_z \cdot z \cdot n} i, \qquad (6.28)$$

where L – length of the path movement by the tool in the feed direction, mm;

l – length of the treated surface, mm;

l_1 – value for tool cutting-in and overrun, mm;

l_2 – additional length for taking test chips. Depending on the mill cutter size – $l_2 = 5 ... 10$ mm;

f_m – feed cutter minute, mm/min;

i – number of passes.

3. The variable part of the cost price C, depending on the cutting modes, is written as:

$$C = A \cdot t_m + (A \cdot t_{ch} + A') \cdot \frac{t_m}{T}, \qquad (6.29)$$

where A – the cost of machine time, cent/min;

A' – cost of the tool, reduced to one period of tool life (depends on the type of tool), cent/ min;

$t_m \approx t_c$ – machine time, min;

t_{ch} – tool change time, min.

The tool change time during milling depends on the number of teeth of the cutter [93-96] and the method of grinding the back surface (without backing, one and two-time backing) and can be written as:

$$t_{ch} = t_y + t_g z \;\; t_{ch} = t_i + t_g z,$$

where t_i – innovation time (to mount the cutter and the machine adjustment), min;

t_g – grinding time for one tooth, min.

Mathematical model of the milling process

The mathematical model in the problem of optimizing cutting modes during milling is the combination of the system of inequalities (6.24) ... (6.27) and the equations of the objective function (6.28) or (6.29).

When using the linear programming method, the desired process model is:

$$\left.\begin{aligned} & x_1 + y_v x_2 \le b_1 \\ & x_1 + y_p x_2 \le b_2 \\ & y_p x_2 \le b_3 \\ & x_2 \ge b_4 \\ & x_2 \le b_5 \\ & x_1 \ge b_6 \\ & x_1 \le b_7 \\ & F_o = x_1 + x_2 \to \max \end{aligned}\right\} B.$$

The optimal values of n_0 and f_{z0} are calculated by formulas (6.10). When using the geometric programming method, the mathematical model in the direct and dual formulation is represented in the form of dependency systems (6.11), (6.12) and (6.13) ... (6.15).

When using the Lagrange multiplier method, the mathematical model includes the Lagrange function and the system of equations (6.16) and (6.17).

Example 4. Linear programming. On a vertical milling machine, process a base type part characterized by the following dimensions: B = 100 mm; l = 300 mm; R_z = 40 μm; part material heat-resistant steel DIN X6 Cr Ni Ti 18 9 (12X18H9T (σ_b = 660 MPa); workpiece – forging; allowance h = 4 mm. Machine vertically milling 6R13F301. Passport data: spindle rotation frequency, min^{-1}: n = 40 ... 2000 (stepless regulation); table feed, mm/min: f_m = 10 ... 2000 (stepless regulation); the greatest feed force: $[P_f]$ = 7550 N; the capacity of the electric motor of the main drive N_m = 7.5 kW; coefficient of performance η = 0.8. End mill, D_m = 150 mm; the number of teeth z = 6; φ = 60 °; material of cutting tip – VK8 hard alloy. Mounted the mill on the limb. Mounted the detail on a table with fastening with bolts and slats with a simple alignment. Detail weight – 5 kg. Small batch production.

Formation of a system of restrictions

1. Constraints on cutting capabilities. Using the dependence (6.24) and reference data [6], we represent in the form of inequality:

$$V = \frac{108 \cdot D_m^{0.2}}{T^{0.32} d^{0.06} f_z^{0.3} B^{0.2}}.$$

We form the constraint explicitly:

$$nf_z^{0.3} \le \frac{318 \cdot 108 \cdot 150^{0.2}}{150 \cdot 180^{0.32} \cdot 4^{0.06} 100^{0.2}} = 43.37.$$

The tool life period T (average value) can be assigned according to the reference manual [30] depending on the type of mill cutter and its diameter. For this case, T = 180 min.

2. The drive capacity constraint of the machine main movement. According to [30], the components of inequality (6.25) take the values:

$$\{C_p = 218; x_p = 0{,}92; y_p = 0{,}78; u_p = 1; q_p = 1{,}15\},$$

and the constraint takes the form:

$$nf_z^{0.78} \le \frac{975 \cdot 10^3 \cdot 7.5 \cdot 0.8 \cdot 150^{1.15} \cdot 2}{218 \cdot 4^{0.92} \cdot 100 \cdot 6 \cdot 150} = 52.98.$$

3. Strength constraint of the feed mechanism. Taking into account (6.26) and data [30]:

$$f_z^{0.78} \le \frac{7550 \cdot 150^{1.15}}{0.89 \cdot 10 \cdot 218 \cdot 4^{0.92} \cdot 100 \cdot 6} = 0.58.$$

4. Parametric constraints on the machine passport:

$$40 \le n \le 2000;$$

$$0{,}00083 \le f_z \le 8{,}33.$$

The formation of the objective function

As the objective function, we choose the machine time spent on milling the plane (l = 300 mm):

$$t_m = \frac{l + \left[0.5 \cdot \left(D_m - \sqrt{D_m^2 - B^2}\right) + 5\right] + l_2}{f_m} \cdot i.$$

After substituting the data of example 4 and [19], we obtain

$$t_m = \frac{300 + \left[0.5 \cdot \left(150 - \sqrt{150^2 - 100^2}\right) + 5\right] + 10}{6 \cdot f_z \cdot n} = \frac{55.68}{f_z \cdot n}.$$

Mathematical model

For the case of face milling, model B will take the following form:

$$\left.\begin{array}{l} x_1 + 0.3x_2 \le 3.77 \\ x_1 + 0.78x_2 \le 3.97 \\ 0.78x_2 \le -0.54 \\ x_1 \le 7.6 \\ x_1 \ge 3.69 \\ x_2 \le 2.12 \\ x_2 \ge -7.9 \\ F_o = x_1 + x_2 \to \max \end{array}\right\} B'.$$

Graphical interpretation and determination of optimum cutting modes

In fig. 6.3, double logarithmic scales depict straight lines describing the inequalities of the system B' the area of possible solutions $\{A'B'C'D'\}$ is selected. The linear function $F_o = x_1 + x_2$ will take a maximum at point C', and the coordinates of this point, $\{f_{z0}; n_0\}$ are the optimal solution to the system B'. They are f_{z0} = 0.497 mm/tooth; n_0 = 53.3 rpm; V_0 = 25.11 m/min; t_{m0} = 2.1 min.

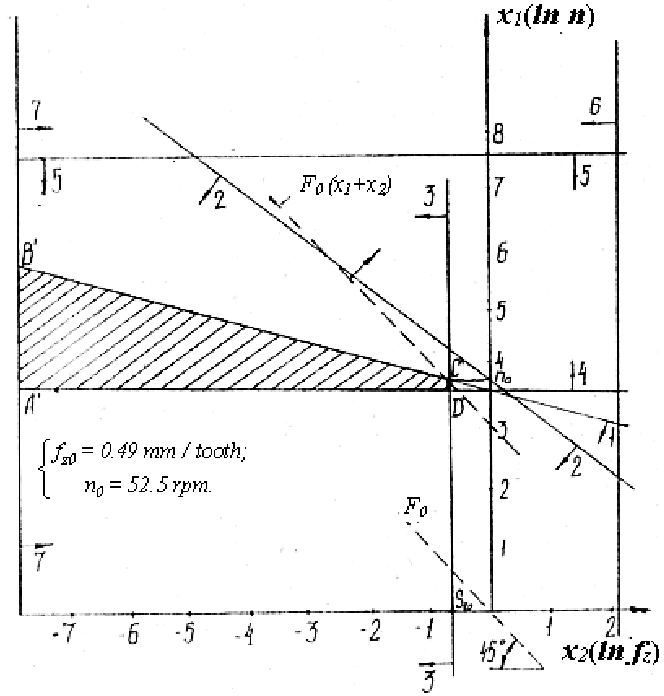

Fig. 6.3. The scheme of finding the optimal milling modes

Example 5. Geometric programming. Determine the optimal cutting modes for face milling (source data borrowed from example 4).

Formation of a system of constraints

Using the result in the previous example, we select two active constraints that determine the optimal value of cutting modes:

Cutting capacity constraint:

$$nf_z^{0.3} \le 43.37.$$

Strength constraint of the machine feed mechanism:

$$f_z^{0.78} \le 0.58.$$

We bring these constraints to the standard form in the GP:

$$0.026 \cdot n f_z^{0.3} \le 1;$$
$$1.724 \cdot f_z^{0.78} \le 1.$$

Assignment the objective function

As an objective function, we consider a variable part of the cost price, depending on the cutting modes. According to the data of [1], the components of the dependence (6.29) take the following values: A = 3.82 cent/min, [86]; t_i = 1.35 min [30]; t_g = 2 min [86]; A' = 0.63 cent/min [86].

The objective function (6.29) when substituting the dependences $T = F(f_z, n)$ and $t_m = F(f_z, n)$ takes the form:

$$C = \frac{3.82 \cdot 55.68}{f_z \cdot n} + \left[3.82 \cdot (1.35 + 2 \cdot 6) + 0.63\right] \cdot \frac{55.68 \cdot n^{3.13} f_z^{0.94}}{0.24 \cdot 10^8 \cdot f_z \cdot n} =$$
$$= 212.7 \cdot f_z^{-1} n^{-1} + 1.19 \cdot 10^{-4} \cdot f_z^{-0.06} n^{2.13}.$$

Mathematical model

A direct statement of the GP problem in face milling is presented in the following form:

Minimize:

$$g_0(n, f_z) = 212.7 \cdot f_z^{-1} n^{-1} + 1.19 \cdot 10^{-4} \cdot f_z^{-0.06} n^{2.13}, \quad (6.30)$$

under the constraints:

$$0.026 \cdot n f_z^{0.3} \le 1; \quad (6.31)$$
$$1.724 \cdot f_z^{0.78} \le 1. \quad (6.32)$$

This task has a first degree of difficulty. We use the Wild technique and, similarly to Example 2, we solve the optimization problem.

First, we transform the statement (6.30) ... (6.32). As a GP problem of zero degrees of difficulty by discarding the term associated with the cost of the tool.

Minimize:

$$C^I = 212.7 \cdot f_z^{-1} n^{-1},$$

with constraints:

$$0.026 \cdot n f_z^{0.3} \le 1;$$
$$1.724 \cdot f_z^{0.78} \le 1.$$

Corresponding dual form:

Maximize:

$$V(w) = \left(\frac{C_{01}}{w_{01}}\right)^{w_{01}} \cdot C_{11}^{w_{11}} \cdot C_{21}^{w_{21}},$$

under the constraints:

$$\begin{aligned} & w_{01} = 1; \\ & n: -w_{01} + w_{11} = 0; \\ & f_z: -w_{01} + 0.3 \cdot w_{11} + 0.78 \cdot w_{21} = 0. \end{aligned} \quad (6.33)$$

The value of dual weights w_i:

$$\{w_{01}=1; w_{11}=1; w_{21}=0.9\}.$$

The value of the objective function is:

$$V(w)=212.7\cdot 0.023\cdot 1.724^{0.9}=8.02 \text{ cent.}$$

From the conditions of invariance, we determine the values of the best cutting modes:

$$\left\{\begin{matrix} f_{z0}=0.497\dfrac{\text{mm}}{\text{tooth}}; n_0=53.3 \text{ min}^{-1}; V_0=25.11\dfrac{\text{m}}{\text{min}}; \\ C_0^I=8,02 \text{ cent.}\end{matrix}\right\}$$

We form the GP model of the first degree of difficulty:

$$C^{II}=212.7\cdot f_z^{-1}n^{-1}+1.19\cdot 10^{-4} f_z^{0.06}n^{2.13}. \qquad (6.34)$$

The constraints on this option coincide with those previously calculated as GP problems of the zero degrees of difficulty (6.31), (6.32).

We define the lower bound of the objective function C^{II} in the statement (6.34), (6.33):

$$C^{III}=212.7\cdot 0.497^{-1}\cdot 53.3^{-1}+1.19\cdot 10^{-4}\cdot 0.497^{-0.06}\cdot 53.3^{2.13}==8.61 \text{ cent.}$$

We calculate the main weights of the terms of the objective function:

$$w_{01}^I=\frac{8.02}{8.61}=0.93;\ w_{02}^I=\frac{0.59}{8.61}=0.07.$$

A new set of dual weights is:

$$\{w_{01}=0.93;\ w_{02}=0.93;\ w_{11}=1;\ w_{21}=0.9\}.$$

We form the conditions of orthogonality and normalization in the statement (6.34), (6.33):

$$n:\ -w_{01}+2.13\cdot w_{02}+w_{11}=0;$$
$$f:\ \ -w_{01}+0.06\cdot w_{02}+0.3\cdot w_{11}+0.78\cdot w_{21}=0;$$

$$w_{01}+w_{02}=1. \qquad (6.35)$$

We choose the dominant terms in system (6.35). These include the first { w_{01} = 0.93}; third { w_{11} = 1}; fourth { w_{21} = 0.9}. Imagine:

$$\{w_{01}=0.93; w_{02}=0.93; w_{11}=1; w_{21}=0.9\}.$$

For all w_i to be positive, the weight w_{02} must be at least 0.32. We express the dual function in a general form as a function of the dual weight w_{02}:

$$V(w_{02})=\frac{\left(\dfrac{212.7}{1-w_{02}}\right)\cdot\left(\dfrac{1-w_{02}}{212.7}\right)^{w_{02}}\cdot 1.19^{w_{02}}\cdot 0.023\cdot 1.724^{0.9}}{w_{02}^{w_{02}}\cdot 0.023^{3.13\cdot w_{02}}\bullet 10^{4\cdot w_{02}}}=$$
$$=7.99-(1-w_{02})^{w_{02}-1}\cdot\left(\frac{0.08}{w_{02}}\right)^{w_{02}}.$$

We use the dichotomy method to determine the optimal values of the weights of the terms and the corresponding value of the objective function:

$$\{w_{02}=0.099;\ V(w_{02})=C_0=8.59 \text{ cent}\}.$$

We determine the optimal cutting modes that minimize the variable part of the cost price, based on the conditions of invariance:

$$212.7\cdot n^{-1}\cdot f_z^{-1}=0.93\cdot 8.59;\qquad (6.37)$$
$$1.724\cdot f_z^{0.78}=1.$$

As a result of solving system (6.37), we obtain:

$$n_0=53.57\ \text{min}^{-1};\ \ f_{z0}=0.497\ \text{mm / tooth};$$
$$V_0=25.25\ \text{m / min};\ f_{m0}=159.75\ \text{mm / min};$$
$$C_0=8.59\ \text{cent}.$$

Example 6. The method of Lagrange multipliers.
The objective function and constraints are of the form:

$$C=212.7\times f_z^{-1}n^{-1}+1.19\cdot 10^{-4}\cdot f_z^{-0.06}n^{2.13}=\alpha_1\times f_z^{-1}n^{-1}+\alpha_2\cdot f_z^{0.06}n^{2.13};$$
$$nf_z^{0,3}\le 43.47=\beta_1;\ \ -\text{tool cutting capabilities}$$
$$f_z^{0.78}\le 0.58=\beta_2.\ -\ \text{feed mechanism strenght}$$

We compose the Lagrange function:

$$L=C+\lambda_1\cdot\left(\beta_1-nf_z^{0.3}\right)+\lambda_2\cdot\left(\beta_2-f_z^{0.78}\right).$$

Take private derivatives:

$$\frac{\partial L}{\partial n}=-\alpha_1 f_z^{-1}n^{-2}+2.13\cdot\alpha_2 f_z^{-0.06}n^{2.13}-\lambda_1 f_z^{0.3}=0;$$
$$\frac{\partial L}{\partial f}=-\alpha_1 f_z^{-2}n-0.06\cdot\alpha_2 f_z^{-1.06}n^{2.13}-0.3\cdot\lambda_1 n f_z^{-0.7}-0.78\cdot\lambda_2 f_z^{-0.22}=0;$$
$$\frac{\partial L}{\partial \lambda_1}=\beta_1-nf_z^{0.3};$$
$$\frac{\partial L}{\partial \lambda_2}=\beta_2-nf_z^{0.78}.$$

Let $\lambda_1\neq 0$ and $\lambda_2=0$. Then we get:

$$\begin{cases}-\alpha_1 f_z^{-1}n^{-1}+2.13\cdot\alpha_2 f_z^{-0.06}n^{2.13}-\lambda_1\beta_1=0;\\ -\alpha_1 f_z^{-1}n^{-1}-0.06\cdot\alpha_2 f_z^{-0.06}n^{2.13}-0.3\cdot\lambda_1\beta_1=0.\end{cases}$$

After conversion and substitution we get:

$$0.7\cdot\alpha_1 f_z^{-1}n^{-1}+0.609\cdot\alpha_2 f_z^{-0.06}n^{2.13}=0\rightarrow f_z^{0.94}n^{3.13}=\frac{-0.7\cdot\alpha_1}{0.609\cdot\alpha_2}.$$

The last expression has no physical meaning because $n>0$ and $f>0$.
Let $\lambda_1=0$ and $\lambda_2\neq 0$. Then we get:

$$\begin{cases}2.19\cdot f_z^{-0.06}n+0.78\cdot\lambda_2\beta_2=0;\\ f_z^{0.78}=\beta_2.\end{cases}$$

Converting this system gives the equation:

$$f_z^{-0.06}n^{2.13}=\frac{-0.78\cdot\lambda_2\beta_2}{2.19}.$$

Since, $\lambda_2 \geq 0$, the last expression has no physical meaning.

Let $\lambda_1 = 0$ and $\lambda_2 = 0$. Then we have:

$$\begin{cases} nf_z^{0.3} = \beta_1 \rightarrow n = \beta_1 f_z^{-0.3} = 43.47 \cdot 0.498^{-0.3} = 53.58 \text{ min}^{-1}; \\ f_z^{0.78} = \beta_2 \rightarrow f_z = \beta_2^{1.28} = 0.498 \dfrac{\text{mm}}{\text{tooth}}. \end{cases}$$

This solution satisfies the initial constraints and affords a minimum of the objective function in the range of admissible values.

7. CALCULATION OF CUTTING MODES ON SOFTWARE EQUIPMENT USING REGULATIONS

The selection of cutting modes $\{V, f, d\}$ and energy characteristics $\{P\}$ on CNC machines and machining centers (MC) is carried out in the following sequence [97].

1. *Assignment of cutting depth d.* For roughing, it should be as high as possible, equal to all or most of the machining allowance *h*. For CNC and MC machines [98-100], the uniformity of the allowance is especially important, which is achieved by precise methods of manufacturing a workpiece or by preliminary grinding of the cast, forged and press forming workpieces. For roughing, worn and lost precision machines with a cyclic and even a numerical control system can be used. When dividing the allowance for semi-finishing in two passes, the following ratio is recommended: $d_1 = (0.6 - 0.75)h$; $d_2 = (0.25 - 0.3)h$ (second pass). When finishing, the cutting depth is assigned depending on the accuracy and surface roughness in the range of 0.5 ... 2.0 mm per diameter, and when processing with a surface roughness of $R_a < 1.25$ – in the range of 0.1 ... 0.4 mm . When assigned *d* to CNC and MC machines, the depth of cut is additionally checked depending on the main angle in the plan (in kinematics, φ_k). When finishing for a contour cutter, usually the angle in plan $\varphi = 90...95^0$ (to completely process the part profile), while roughing for cutters $\varphi < 90^0$. Besides, the depth of cut should be such that the working length of the cutting edge l_c satisfies the condition $l_c = 0.75 \cdot l$ where *l* – the size of the indexable insert.

2. *The assignment of the feed.* For turning on non-automated equipment, the feed *f* is assigned according to the tabular data [30]. When

roughing f is assigned the maximum based on the rigidity and strength of the machine system elements, the machine drive capacity and other factors. During machining finishing, it is selected depending on the required degree of accuracy of the finished product and the roughness parameters of the processed surface. The level of technological support affects the value of selectable feed for CNC machines [101-103]. It is characterized by the presence of incoming quality control, hardness and strength of the cutting inserts, the implementation of preliminary roughing and heat treatment, the presence of rigid equipment, auxiliary tools, etc. The choice of coefficient K_f depends on the level of technological support (table 7.1).

Table 7.1

Coefficients affecting feed selection

Technological level providing	K_f	K_T
High	1.1…1.4	0.4…0.9
Medium	1	1
Low	0.5…0.9	1.1…2.0

Nominal feed f_n, selected according to the machine-building normals [30], can be increased or decreased depending on the level of technological support. Tables for selecting feeds for roughing and finishing turning and boring are given in [37; 104].

After selecting the normal feed rate, verification calculations are made. For example, rough feeds are checked by axial force (should not exceed the force allowed by the feed mechanism), by the strength of the holder and the cutting insert of the cutter, etc. For verification calculations, the following formulas can be used:

The feed f_{hs} allowed by the strength of the tool holder:

$$f_{hs} = \sqrt[y_p]{\frac{BH^2[\sigma_{ben}]}{6C_{pz}d^{X_{pz}}l_c k_{pz}}}. \qquad (7.1)$$

The feed f_{si} allowed by the strength of the cemented carbide insert:

$$f_{si} = \sqrt[y_{pz}]{\frac{34\cdot C^{1.25}(\sin 60^0/\sin\varphi)^{0.8}}{C_{pz}d^{X_{pz}-0.77}k_{pz}}}. \qquad (7.2)$$

The feed f_{sf} allowed by the strength of the machine feed mechanism:

$$f_{sf} = \sqrt[y_{pz}]{\frac{P_m}{C_{px}d^{X_{px}}k_{px}}}. \qquad (7.3)$$

The convention notations used in dependencies (7.1)…(7.3) are deciphered in chapter 2).

When finishing turning, it is advisable to check the stiffness of the workpiece and cutter:

The feed $f_{dr.}$ allowed by the rigidity of the machined detail:

$$f_{dr.} = \sqrt{\frac{f_b E_d \mu \Im}{1.1\cdot C_{py}d^{X_{py}}l_d^3 k_{py}}}. \qquad (7.4)$$

The feed allowed by the rigidity of the cutter:

$$f_{cr} = \sqrt{\frac{f_c E_c BH^3}{4\cdot C_{pz}d^{X_{pz}}l_c^3 k_{pz}}}, \qquad (7.5)$$

where f_{cr} – the arrow of the cutter bending, which is equal to 0.1 and 0.05 mm for rough and clean turning, respectively; E_c = 200 GPa – modulus of elasticity of the tool holder; *B, H* – the width and height of the tool holder; l_c – the length of the cutter overhang (the remaining conventional notation are deciphered in chapter 2).

Accepted after performing verification calculations, the feed is specified according to the machine passport. In this case, take the closest feed available on the machine. It is allowed to accept the nearest large feed if it exceeds the normative by no more than 10%. When choosing a feed for cutting tungsten oxide ceramic inserts (WOC 60, etc.) at the stage of cutting-in and exit of the cutter, as well as during the transition of the cylinder – end face in the case of contouring, it is recommended to accept the feed within 50% of the table value, and then increase to 125% [37]. An alternative to analytical verification calculations is to use tabular of maximum allowable feeds. So, when turning parts made of C45 (steel 45), the maximum feed rates, constrained by the strength of the cemented-carbide insert with a plane angle $\varphi = 45^0$, are given in the table. 7.2.

Table 7.2

Maximum feed rates

Thickness insert, C_i, mm	Depth of cut, *d*, mm			
	4	7	13	22
4	1.3	1.1	0.9	0.8
6	2.6	2.2	1.8	1.5
8	4.2	3.6	3.0	2.5
10	6.1	5.1	4.2	3.6

Notes:1. When machining details from cast iron, the feed value must be multiplied by a factor of 1.6.

2. At $\varphi = 30^0$, the feed value should be multiplied by a correction factor of 1.4, at $\varphi = 60^0$ – by 0.6, at $\varphi = 90°$ – by 0.4.

3. *The choice of the tool life period T*. For calculating the cutting conditions on universal equipment, standard tool life T_s values are accepted taking into account economic factors and production experience of operating the tool. A recommendation of the type: "the average value T_s for single-tool machining by turning is 30 ... 60 min" [30] does not suit designers of processing technology on CNC machines. The main feature of the choice of cutting modes for CNC machines is the justification of economically feasible tool life T_e (T_{pr}, T_c) i.e. such tool life, which for the given cutting conditions provides the highest productivity T_{pr} and minimum machining cost T_c [97]:

$$T_e = \left(\frac{1}{m} - 1\right)\left(t_{ch} + \frac{\theta_t + \theta_g}{E}\right),$$

where *m* – an indicator of relative tool life [30]; t_{ch} – tool change time, min; θ_t – the costs associated with the operation of the tool for one period of tool life, cent; θ_g – the costs associated with the regrinding of the tool, cent; E – the cost of one machine-minute, including the wage of the worker with accruals, cent. Specific cost indicators are given in [79].

The value θ_t takes into account only the cost of the cutter insert, related to the frequency of its use, to the number of angles if the plate is single-acting (type *R, M*), or to the double number of angles if the plate is double-acting (type *N, A, F, G*).

Let's consider examples of θ_t definitions. For a brazed cemented-carbide tool with a cost of 5 EUR that can withstand 10 regrind, the value θ_t is 50 cents, and if the cost of a one-sided tetrahedral insert in a cutter tool is 40 cent, the value θ_t is 10 cent. For tools with multi-edged non-grindable plates, the value $\theta_g = 0$.

If we are talking about the operation of CNC machines and MC, the use of a cheap tool is impractical. Therefore, the normative period of economic tool life T_{en} should be chosen in the range of T_{en} = 20 ... 40 min [97] for the average level of technological support. High and low levels of collateral are taken into account by the coefficient K_T (see table. 7.1).

With multi-tool processing, the tool life T_{tm} period should be increased depending on the number of simultaneously working tools: $T_{tm} = T_{el} K_{Tm}$, where T_{el} – economic tool life of the limiting tool. The values of the coefficient for change of the tool life period during multi-tool processing takes on value:

Number of working tools	1	2	3	4	5	6
K_{Tm}	1.0	1.7	2.0	2.5	3.0	4.0

When turning complex parts for a more detailed calculation of the tool life period, it is necessary to take into account the variability of feed and cutting speed within one processing cycle, the difference in load and reliability of individual tools in adjustment, and other factors [37]. The calculated tool life period $T_{cal} = T_s \cdot K_1 \cdot K_2 \cdot K_3 \cdot K_4$, where K_1 – coefficient taking into account the change in the direction of movement of the cutter relative to the surface to be treated; K_2 – coefficient taking into account the change in feed and cutting depth during the tool life period (table. 7.3); K_3, K_4 – coefficients that take into account the different loading and reliability parameters of tools in multi-tool adjustment (Table 7.4).

Table 7.3

Coefficients affecting on T_{cal}

Type of cutter	Type of chuck	K_1	K_2
Straight turning Cutter for rough	Autolock chuck	0.89	0.86
	Between centers	1.0	0.8
Straight turning Cutter for rough and facing cutter	Autolock chuck	0.83	075
Facing cutter for roughing machining	Autolock chuck	1.0	0.86
Straight turning contour	Between centers	0.75	0.75
Boring contour	Autolock chuck	0.96	0.75

Table 7.4

Coefficients of load and reliability

Coefficient	Number of tools in adjustment							
	1	2	3	3.5	4	5	5.5	6 and more
K_3	1.0	1.3	1.5	1.65	1.7	1.85	1.9	1.95
K_4	1.0	1.05	1.1	1.15	1.2	1.25	1.25	1.3

4. *The choice of cutting speed V, m/min*, depends on the economic tool life T_e. The depth of cut and the feed are chosen to be maximum taking into account the constraints, and the cutting speed is such as to ensure the tool life economic $V = C_v / T_e^m$, where C_v – coefficient reflecting the influence of the material being processed, the geometry of the cutting part of the cutter and other parameters not taken into account in this dependence.

In some sources [37; 79] the cutting speed is selected according to the tables, and corrected by entering a coefficient K_{TV} [79] depending on the ratio T_e/T_s and coefficients K_1 and K_2, according to the table. 7.3.

5. *The calculation of the spindle speed n, rpm*, is performed using the ratio $n = 1000 \cdot V / \pi D$, where D – processing diameter. The obtained value is specified according to the passport of the machine n_p and take the nearest smaller of the available on the machine. Exceed the design frequency is allowed no more than 10%.

6. The actual speed V_a, m/min, is determined by the adopted spindle speed: $V_a = \pi D n_p / 1000$.

7. *Checking the selected cutting mode for the machine power N_m*, is carried out only for roughing by checking the fulfillment of the ratio $N_c \le N_m \eta$, where η – efficiency of the main cutting movement drive; N_c – cutting capacity, determined by the tables of [1] or by the formula:

$$N_c = \frac{P_z}{1020 \cdot 60} = \frac{10 \cdot C_{pz} f^{y_p} d^{x_p} V^{n+1} k_{pz}}{1020 \cdot 60}.$$

If it turns out that the capacity of the machine electric motor on which the processing is to be carried out is insufficient, it is necessary to choose a more powerful machine. If this is not possible, it is necessary to reduce the issued speed or feed by moving to the next smaller one. In this case, it is necessary to dwell on such a final decision, in which the product $n \cdot f$ will be the largest since in this case the machine processing time will be the shortest.

When reducing the cutting speed in checking calculations, you can use the method proposed in [105]. It comes down to determining the coefficient of cutting speed change K_V [105] concerning $N_c / N_m \eta$ and calculating the $V_{mp} = V \cdot K_V$ more precise.

In the case of multi-tool processing, the calculation is based on the total capacity $\sum N_c$, kW, calculated by the cutting capacity for each tool with the subsequent summation of the capacity of simultaneously working tools.

8. *Determination of machine time t_m, min.* For every pass:

$$t_m = \frac{l + l_1 + l_2}{n \cdot f},$$

where l – length of the surface to be treated; l_1, l_2 – the length of the cutting-in and overrun, respectively, determined by the table [63].

Machine time spent on the operation as a whole,

$$t_c = \frac{(l + l_1 + l_2) i}{n \cdot f},$$

where i – given a number of passes.

Along with the traditional step-by-step selection and calculation of cutting modes according to the standards, more sophisticated methods have also been developed, which are also based on industrial normals.

Consider the method of calculating turning modes. As the objective function, we take the cost price of processing in one pass:

$$C = A\frac{\pi D l}{1000 \cdot Vf} + At_{ch}\frac{\pi D l}{1000 \cdot Vf} V^{n_1 - 1} f^{n_2 - 1} + B\frac{\pi D l}{1000 \cdot K} V^{n_1 - 1} f^{n_2 - 1} =$$
$$= X \cdot V^{-1} f^{-1} + Y \cdot V^{n_1 - 1} f^{n_2 - 1} + Z \cdot V^{n_1 - 1} f^{n_2 - 1},$$

where $T = \frac{K}{V^{n_1} f^{n_2}}$; $K = \frac{C_v}{d^{y_v}}$; $X = A\frac{\pi D l}{1000}$; $Y = A\frac{\pi D l}{1000 \cdot Vf}\frac{t_{ch}}{K}$;

$Z = \frac{B \pi D l}{1000 K}$.

Used characters in formulas are deciphered in chapters 1 and 2).

A necessary condition for the minimum cost price is the equal to zero gradients of the function C, i.e. all private derivatives:

$$\frac{dC}{dV}=0;\ \frac{dC}{df}=0.$$

From these equations it follows:

$$\frac{dC}{dV}=-X\frac{1}{fV^2}+Yf^{n_2-1}(n_1-1)V^{n_1-2}+Zf^{n_2-1}(n_1-1)V^{n_1-2}=0;$$

$$-X+Y(n_1-1)V^{n_1}f^{n_2}+Z(n_1-1)V^{n_1}f^{n_2}=0; \quad (7.6)$$

$$\frac{dC}{df}=-X\frac{1}{Vf^2}+YV^{n_1-1}(n_2-1)f^{n_2-2}+ZV^{n_1-1}(n_2-1)f^{n_2-2}=0;$$

$$-X+Y(n_2-1)V^{n_1}f^{n_2}+Z(n_2-1)V^{n_1}f^{n_2}=0; \quad (7.7)$$

Thus, equality must be satisfied:

$$\frac{X}{Y+Z}=(n_1-1)V^{n_1}f^{n_2}=(n_2-1)V^{n_1}f^{n_2}.$$

This equation is solvable only if the exponents are equal for V and f. Since this is practically impossible, the function C does not have a single minimum.

The graphs of functions (7.6) and (7.7) are shown in fig. 7.1.

The axis on which the machining cost price is set aside is perpendicular to the V-f plane. For turning, the condition $n_2 < n_1$ must be met, therefore, for any feed value f, the cutting speed satisfying equation (7.6) will be less than the cutting speed calculated from equation (7.7).

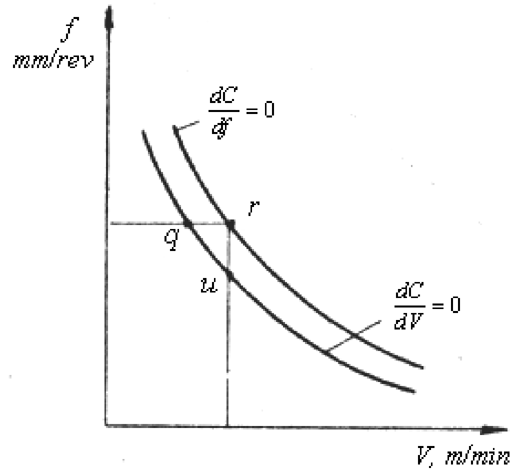

Fig. 7.1. Level lines by the criterion of minimum cost price

In [5], it was shown that the prime cost at a point ***q*** is lower than at a point ***r***, and at a point, ***r*** is lower than at ***u***. Therefore, the cost price of the part decreases with increasing feed. The cutting modes corresponding to the permissible minimum cost price of a passage should be selected from the maximum allowable feed and cutting speed calculated according to equation (7.6), which can be represented in the following form:

$$\frac{K}{V^{n_1}f^{n_2}}=(n-1)\left(\frac{At_{ch}+B}{A}\right)=T_V,$$

where T_V – the tool life corresponding to the minimum of the part cost price.

Optimum speed corresponding to the tool life:

$$V = \frac{K^{\frac{1}{n_1}}}{T_V^{1/n_1} f^{n_2/n_1}},$$

where f – maximum possible feed rate value.

Thus, the procedure for calculating the optimal cutting modes will be as follows.

1. Choose the maximum feed f based on the capabilities of the machine.

2. We calculate the cutting speed V.

3. The calculated speed is compared with the maximum speed of the machine V_{max}. If, $V < V_{\max}$ correction is not required. Otherwise, you should take the maximum speed provided by the machine.

4. We take into account constraints on the maximum machine capacity. For turning, the cutting speed V_m allowed by the machine is:

$$V_m = \left(\frac{6120 \cdot N_c^m K_t}{C_{pz} d^{x_{pz}} f^{y_{pz}} k_{pz}} \right)^{1/(1+n)}.$$

Here coefficient K_t reflecting the influence of the material of the cutting tool on the cutting speed: K_t = 1.25 – when machining with a cemented-carbide tool and K_t = 1 when machining with high-speed steel of tool.

The cutting speed is corrected according to the value K_t.

5. Check the constraints on the maximum cutting force. Turning this force is determined by the well-known formula [30]. If the cutting force is greater than the value allowed by the feed mechanism of the machine, it is necessary to reduce the feed or choose another machine.

6. We calculate the feed allowable from the conditions for ensuring the required roughness. The height of the micro-irregularities can be

approximately determined from the relation $R_z = f^2/8r$, where r – the radius at the tip of the cutter.

Therefore, for a given finishing surface and tool geometry, there is a feed limit value. If it is less than a previously determined value, then the feed is taken $f = \sqrt{8rR_z}$, and the cutting speed is calculated according to equation (7.6).

7. The last constraint is caused by the stepwise nature of the change in feed and speed. The cutting modes are finally determined by finding the maximum allowable feed value according to the machine passport and calculation cutting speed V. Then select such a speed that provides a real speed that is as close as possible to the calculated value.

Consider an example of the implementation of the methodology for calculating cutting modes using standards.

Initial data: detail – shaft, D_1 = 200 mm; length of detail l_d = 1000 mm; machine – 16K20F3 (f_{max} = 2.8 mm/rev; n_{min} = 12.5 rpm; n_{max} = 2000 rpm; N_m = 10 kW, η = 0.65); cutting depth d = 3 mm, tool change time t_{ch} = 0.08 min.

We calculate from the condition of minimum cost price [63].

1. Choose the maximum feed based on the capabilities of the machine: $f = f_{max}$ = 2.8 mm/rev

2. Using the formula (7.6), we calculate the cutting speed:

$$V = \sqrt[n_1]{\frac{X}{(Y+Z)\cdot(n_1-1)\cdot f^{n_2}}} =$$

$$= \sqrt[3]{\frac{373}{5.21\cdot 10^{-12} + 2192\cdot 10^{-12}\cdot(5-1)\cdot 2.8^{2.25}}} = 84.0 \text{ m/min},$$

where

$$X = \frac{A\pi D_1 l_d}{1000} = \frac{0.594\cdot 3.14\cdot 200\cdot 1000}{1000} = 373;$$

$$Y = X\cdot\frac{t_{ch}}{K} = 373\cdot\frac{0.08}{5.73\cdot 10^{12}};$$

$$n_1 = \frac{1}{m} = \frac{1}{0.2} = 5;\ n_2 = \frac{y}{m} = \frac{0.45}{0.2} = 2.25;$$

A = 0.594 cent/min [30]; B = 20 cent; C_V = 420.0; x = 0.15;
y = 0.45; m = 0.2 [30];
4. Compare the obtained speed with the permissible machine speed:

$$V_{\max} = \frac{\pi D_1 n_{\max}}{1000} = \frac{3.14\cdot 200\cdot 2000}{1000} = 1256 \text{ m/min};\ V_{\min} = \frac{\pi D_1 n_{\min}}{1000} =$$
$$= \frac{3.14\cdot 200\cdot 12.5}{1000} = 7.85 \text{ m/min}.$$

Since $V_{\min} < V < V_{\max}$, we take V = 84 m/min.

4. Check the speed limit on the power of the machine [63]. To do this, we calculate the cutting speed:

$$V_m = \left(\frac{6120\cdot N_c^m\cdot K_t}{C_{pz}\cdot d^{x_{pz}}\cdot f^{y_{pz}}\cdot K_z}\right)^{1/(1+n)} = \left(\frac{6120\cdot 8.0\cdot 1.25}{300\cdot 3^1\cdot 2.8^{0.75}\cdot 0.84}\right)^{1/(1-0.15)} = 72\ m/\text{min},$$

where $N_c^m = 10\cdot 0.8 = 8.0$ kW; K_t = 1.25 – when machining with carbide cutter;

$C_{pz} = 300;\ x_{pz} = 1.0;\ y_{pz} = 0.75;\ n = -0.15$ [30];

$K_{pz} = K_{mp}\cdot K_{\gamma p}\cdot K_{\varphi p}\cdot K_{\lambda p}\cdot K_{tp} = 0.84;\ K_{mp} = 0.84$ [30];

$K_{\gamma p} = K_{\varphi p} = K_{\lambda p} = K_{tp} = 1$ [30].

Since $V > V_m$, we accept $V = V_m = 72$ m/min.
Therefore, the optimal mode V = 72 m/min, f = 2.8 mm/rev.
5. Determine the cutting force:

$$P_z = C_{pz} d^{x_{pz}} f^{y_{pz}} V^n K_z = 300\cdot 3^1\cdot 2.8^{0.75}\cdot 72^{-0.15}\cdot 0.84 = 860 \text{ N}.$$

6. Determine the tool life T_V (effective):

$$T_V = (n_1 - 1)\cdot\left(\frac{A\cdot t_{ch} + B}{A}\right) = (5-1)\cdot\left(\frac{0.594\cdot 0.08 + 20}{0.594}\right) = 135 \text{ min}.$$

7. Determine the height of the irregularities: $R_z = f^2/8r$, where r = 1.5 – the radius at the tip of the cutter; $R_z = 2.8/8\cdot 1.5 = 0.65$ mm.

8. Determine the machining cost price:

$$C = A\cdot t_m + A\cdot t_{ch}\cdot\frac{t_c}{T_V}) + B\cdot\frac{t_c}{T_V},$$

where

$$t_m \approx t_c = \frac{\pi\cdot D_1\cdot l_d}{1000\cdot V\cdot f} = \frac{3.14\cdot 200\cdot 1000}{1000\cdot 72\cdot 2.8} = 3.11 \text{ min}.$$

For this case, the value of the machining cost price is equal to:

$$C = 0.594\cdot 3.11 + 0.594\cdot 0.08\cdot\frac{3.11}{135} + 20\cdot\frac{3.11}{135} = 2.31 \text{ cent}.$$

CONCLUSION

The introduction of the toolkit for the optimal design of technological processes suggests the possibility of using developed methods and algorithms for finding the most rational modes of machining engineering products. In the search process, the designer evaluates various approaches to solving this problem, which is fundamental for effective engineering production. For this assessment, such aspects as the choice of performance criteria and the formation of the objective function, the formation of the system of the most important technical constraints, the implementation of various decision algorithms depending on the specifics of the problem being solved are important.

In the process of researching the optimization of cutting modes, the following results were obtained.

1. The optimization problem has been formulated as a procedure for determining the optimal cutting conditions, implemented by mathematical programming methods, which, depending on the type of constraints and objective function, belong to one of the following classes: linear, nonlinear, discrete, dynamic and stochastic programming.

2. An analysis of various criteria for the efficiency of the cutting process is presented, the inconsistency of their simultaneous achievement is shown, and a procedure for finding optimal solutions on a selected subset of the acceptable values of the cutting mode elements is proposed, which in its properties would be similar to the Pareto set. If the range of admissible values is limited by a closed curve of arbitrary shape, then optimal solutions should be sought at the boundary of the range of admissible values, between the maximally scattered points of tangency of

the functions level lines $\varphi_1 = V \cdot f$ and functions $\varphi_2 = V^m \cdot f^n$ with the specified boundary.

3. A software-methodological complex for solving optimization problems by the method of geometric programming (MGP) has been developed. The effectiveness of this method is shown, especially for solving problems of a nonlinear type, when all the components of the optimization problem can be expressed quantitatively in the form of generalized positive polynomials called posynomials. In this case, the constructive moment is finding, first of all, the extremum of the objective function and the relative contribution of each component to its value, and then the optimal values of the variable parameters X_j^*. The availability of information on the relative contribution of various components to the optimality of the design decision makes it possible to identify the direction for improving the technological systems of machining.

4. For the case of large-dimensional problems, when the system of linear equations does not have a unique solution, computational procedures are implemented in this monograph that reduces such problems to MGP of zero degrees of difficulty. It presents techniques for combining some members of the objective function based on their weights, as well as methods for partial invariance that take into account the dominant terms of the objective function. These techniques must allow you to implement a step-by-step improvement of the initial solution by replacing the dominant terms.

5. At the same time, when the proposed methods of reducing the MGP problem to the zero degrees of difficulty are not applicable if there is no initial solution, needed to evaluate dual variables, the author has proposed a procedure for solving problems of the first degree of difficulty. To implement this procedure, a dual problem is compiled, which is considered as optimization and is solved by searching for the extremum of the only redundant, dual variable values.

6. Three effective methods for numerically solving design problems using the MGP of the first degree of difficulty, reducing the overall complexity of the implementation of optimization calculations, are presented. The use of the partial invariance method in problems of optimizing cutting modes is considered using the example of a typical problem of single-tool cutting processing. The mechanism for implementing this method is reduced to bringing optimization problems to zero degrees of difficulty and consists of iteratively using the property of geometric inequality at a minimizing point to search for optimal (sufficiently close to optimal) values of the controlled variables for the design problem. In the process of solving the difficulties that were associated, firstly, with the need to check compatibility conditions, and, secondly, with a multi-step analysis of different sizes optimizing parameters. To research two-pass turning, the "Wild method" was applied, with the help of which a system of linear equations is solved for the least significant tool component (the dual-use equation system does not have a single solution.

7. The author of the monograph proposes to use the express-procedure of MGP for three-parameter optimization, which allows you to build and research the cost price function of two-pass processing C_2 from the cutting depth on the finishing pass. In this case, the property of constancy of the posynomial terms relationship of the in the MGP problem is used when changing the parameters that determine the constraints imposed. This leads directly to the calculation of the lower boundary of the new minimizing objective function. An experiment was performed in the work and the values of the objective function and cutting elements were calculated in the range of the depth changes of the finishing pass t_2 from 2.5 to 0.05 mm using the classical GP technique and express technique. Analysis of the output data indicates an acceptable approximation of express estimates. The obtained function $C_2 = f(t_2)$ is convex, monotonically decreasing in the direction of the t_2 axis and does not have a global extremum in the entire domain of definition. The lower limit of the cutting depth t_2 will coincide with the calculated minimum finishing allowance Δ, which for this case is Δ = 0.138 mm. The range of changes in depth t_2 at which the search for optimal modes of two-pass processing is carried out is revealed. At the lower limit of this range, the cost price values of two-pass and single-pass machining are comparable. Besides, in a wider range, there is one extreme point of maximum cost (t_2 = 1.3 mm), which should be avoided when assigning cutting depths.

8. The work presents a program-methodological complex for solving optimization problems by the method of Lagrange multipliers (MLM). This method of analytical research of an extreme technological problem makes it possible to determine a qualitative picture of the behavior of the optimal solution when the structure and parameters of the problem change, and is also effectively used in the development of human-machine procedures for searching for extrema, which use analytical tools to analyze the results of numerical solutions of the initial optimization problem.

9. Five alternative options have been obtained for rough turning using MLM, and the set of optimal solutions lies on the border of permissible values, and only inequality $f \le f_{\max}$ is taken into account from parametric constraints. The choice of the best option is carried out by comparing the solutions obtained by the value of the objective function.

10. For finishing turning, when the tool life should be a multiple of the cutting time when machining one part and active force constraint P_z^*, a technique is proposed for introducing an additional equation $T = k \cdot t_c$, that reflects the number of parts processed during the tool life. When jointly solving these equations, two pairs $< V_1, f_1 >$ and $< V_2, f_2 >$ can be determined and between which the optimum point of the cutting modes $< V_0, f_0 >$ lies on the boundary line P_z^*. Such a problem arises when it is

necessary to tool change and the need to strictly comply with the constraint on the cutting force. In this case, the designer is faced with the requirement to reduce the tool life, leaving the cutting time unchanged, or to increase the cutting time, leaving the tool life unchanged.

11. A new approach to optimizing two-pass processing using MLM is proposed. In the case when force constraints are active during two-pass processing, the cutting depth in the first pass is determined using the analytical dependence derived in the monograph, otherwise, the cutting depth in the first pass is accepted as the maximum allowable for technological reasons. It is proved that the obtained optimal distribution of allowance for passages afford the maximum objective function of cost price. Based on the researches, it is confirmed that two-pass processing can be more economical than single-pass processing, especially for the case when it is necessary to obtain a surface with a low degree of roughness on a machine with the main movement low drive power. Moreover, numerical data were obtained on the effect of changes in the allowable cutting force (in various ranges from 2000 to 3000 N) and the depth of cut on the value of the machining cost price. And finally, an analytical condition for the "profitability" of two-pass processing is proposed instead of a single-pass with active force constraints.

Since the machining error caused by fluctuations in the depth of cut due to the tolerance on the original size is 0.1 ... 0.2 of the total machining error, the proposed procedure for calculating the error of the intermediate machining (for example, the accuracy of the size obtained on the first pass), which ensures a minimum cost price.

12. The effectiveness of MML tools is illustrated in this monograph by solving the problem of optimizing three-pass processing. When at each pass the force constraint is active and the stationary point of the cost price objective function is a minimum point, it is proved that the optimal distribution of allowance between passes is determined by equality $t_1 = t_2 = t_3$. Otherwise, when type constraints $f_i \le f_{i\max}$ are active, a

different distribution of the allowance is found during three-pass processing. To do this, an analysis of a stationary point on the nature of the extremum is carried out and using the quadratic form of the Lagrange function, it is shown that it is always the maximum. Consequently, it is proposed to look for a minimum cost price with limited allowance distributions.

Of interest is the consideration of two alternative options for the distribution of allowances during three-pass machining, which differs in the adoption of acceptable values of the cutting depth on the first pass (1_{st} option) and on the third (2_{nd} option). The inequality is determined when the first option is more economical than the second when the relationship between the allowable values of the cutting forces is satisfied: $P_{1\lim} \ge P_{2\lim} > P_{3\lim}$.

13. The monograph presents a program-methodological complex for solving multicriteria optimization problems by linear and multiplicative convolution methods of efficiency criteria, consecutive climb-down and the "ideal point". Each of them has been tested experimentally and allows using multi-criteria optimization for various types of processing. The problem of finding the optimum is formulated for many performance criteria that are contradictory and attain a maximum at various points in the acceptable alternatives set.

14. The problem is solved by two criteria of efficiency: productivity and machining cost price by the method of multiplicative convolution. Moreover, the efficiency criterion thus transformed was presented in the form of the ratio of the maximized criterion to the minimized criterion with the only constraint on the main component of the cutting force. The values of the objective function and optimal cutting modes using the MLM method are obtained.

15. The method of consecutive climb-down to optimize finishing turning according to the criteria of minimum machine time and minimum detail bending deflection arrow is used. When solving the optimization

problem, linear and geometric programming methods, as well as a numerical dichotomy method were used. The linear programming method was used when considering the task of finding the minimum shaft bending deflection arrow with the constraint on the cutting capabilities of the tool and the acceptable value of surface roughness. MGP of the zero degree of difficulty in this formulation is not applicable, due to the appearance of negative dual weights, which contradicts the initial restriction of the GP method. At the same time, the task of finding the minimum machining time is solved by MGP of the first degree of difficulty. A feature of the solution algorithm is integration with the method of numerical optimization (dichotomy, golden ratio, or Fibonacci numbers), which gives a solution that is reasonably close to the optimal (suboptimum) using standard optimization programs for unimodal functions.

As calculations have shown, the desire to minimize machine machining time and bending deflection arrows leads to two sets of optimized parameters. To find a compromise option for the implementation of the machining modes, a procedure for selecting climb-down is proposed (for example, by comparing machine time obtained as a result of single-criterion optimization with a tabulated time value according to the standards).

16. A methodology and algorithms for multicriteria optimization by the “ideal point” method have been developed, when the result obtained corresponds to extreme values of the criteria at the same time. It is shown how the reduced differences between the achieved and extreme values are formed for the maximized and minimized criteria for the case of a three-criterion problem related to minimizing the cutting capacity, detail bending deflection arrow and machine time for shaft roughing machining.

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